General Physics 1 (PHYS1113)

Academic year 1447 - 2

Chapter 1

Measurement

Measuring Things

- To measure any physics quantity, we need magnitude and unit
- For example: the full day is 24 hours
- Each unit must be compared to a standard
- There are 3 *Base Quantities* (Length, Time, Mass).
- We then define all other physical quantities in terms of these *base quantities* and their standards (called *base standards*).
- For example: volume is length × length × length, density is the ratio of mass to volume (volume is length × length × length) …
- Base standards must be both accessible and invariable

Table 1-1 Units for Three SI Base Quantities

Quantity	Unit Name	Unit Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

The Physical Quantities

There are two types of Physical Quantities

1

Basic Physical Quantity

- **Mass** denoted as **M**
- **Length** denoted as **L**
- **Time** denoted as **T**

2

Derived Physical Quantity

- It is any quantity that can be expressed by the basic quantity, such as area, volume, density, force, velocity, etc.

Systems of units

There are three common systems of units

1. International system of units (SI units)
2. British system of units (UK units)
3. Gaussian system of units (cgs units)

System			
Quantity	**SI**	**British**	**CGS**
Length	Meter (m)	Foot (ft)	Centimetre (cm)
Time	second (s)	Second	second
Mass	Kilogram (kg)	Pound_mass (lbm)	Gram (g)
Force	Newton (N)	Pound_force (lbf)	Dyne

The International System of Units (SI Units)

- The *International System* of units (abbreviated as SI units) has only seven quantities that are chosen internationally as *base quantities*. These units are used to *derive* any other unit in science.
- In this course we need only these three base quantities: length, time, mass to derive all other units we will study.

length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

For example:

$$\text{Speed} = \frac{\text{Length}}{\text{Time}}$$

Derived quantity — *base quantities*

$$\therefore \text{Unit of speed} = \frac{\text{unit of length}}{\text{unit of time}} = \frac{m}{s}$$

Table 1-1 Units for Three SI Base Quantities

Quantity	Unit Name	Unit Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

Prefixes for SI Units

- To express the very large and very small quantities we often run into in physics, we use *scientific notation*, which employs powers of 10. In this notation,

  $3\ 560\ 000\ 000\ \text{m} = 3.56 \times 10^{9}\ \text{m}$

  $0.000\ 000\ 492\ \text{s} = 4.92 \times 10^{-7}\ \text{s} = 492 \times 10^{-9}\ \text{s}$

- When dealing with very large or very small measurements, we use the prefixes listed in Table 1-2, For example:

  1.27×10^{9} meter = 1.27 gigameter = 1.27 Gm

  2.35×10^{-9} s = 2.35 nanoseconds = 2.35 ns

Table 1-2 Prefixes for SI Units

Factor	Prefix[a]	Symbol
10^{24}	yotta-	Y
10^{21}	zetta-	Z
10^{18}	exa-	E
10^{15}	peta-	P
10^{12}	tera-	T
10^{9}	**giga-**	**G**
10^{6}	**mega-**	**M**
10^{3}	**kilo-**	**k**
10^{2}	hecto-	h
10^{1}	deka-	da
10^{-1}	deci-	d
10^{-2}	**centi-**	**c**
10^{-3}	**milli-**	**m**
10^{-6}	**micro-**	**μ**
10^{-9}	**nano-**	**n**
10^{-12}	**pico-**	**p**
10^{-15}	femto-	f
10^{-18}	atto-	a
10^{-21}	zepto-	z
10^{-24}	yocto-	y

Changing Units

- We often need to change the units in which a physical quantity is expressed.
- We multiply the original measurement by a **conversion factor**
- For example, because 1 min and 60 s are identical time intervals, we have:

1 min = 60 s ← *Conversion factor between min and s*

- If we need to convert 2 min to seconds:

$$2\ \text{min}\left(\frac{60\ s}{1\ min}\right) = \frac{2\ \text{min}\times 60\ s}{1\ min} = 120\ \text{s}$$

- Another example: we need to change the quantity 15 m/s to its equivalent in cm/min:

Conversion factors are: 1 min = 60 s and 1 m = 100 cm, so,

$$15\ \frac{m}{s}\left(\frac{100\ cm}{1\ m}\right)\left(\frac{60\ s}{1\ min}\right) = \frac{15\ m\times 100\ cm\times 60\ s}{1\ s\times 1\ m\times 1\ min} = 90000\ \text{cm/min}$$

- By using this method, we can convert any unit to any other unit (even if we are not familiar with these units)

<u>Example 1</u>: The speed of a car is 35 m/s. What was this speed in km/h?

<u>Solution:</u>

Conversion factor:
1 km = 1000 m and 1 h = 3600 s

The speed is $v = 35\ \frac{m}{s} = 35\ \frac{m}{s} \times (\frac{1km}{1000\ m}) \times (\frac{3600s}{1h})$

$$= \frac{35 \times 1 \times 3600}{1000 \times 1} \frac{km}{h} = 126\ km/h$$

Notes on unit conversion:

If there is a power in the unit, you must use the same power for the conversion factor.

1 m = 100 cm

$1\ m^2 = (100)^2\ cm^2 \Longrightarrow 1\ m^2 = (10\ 000)\ cm^2$

$1\ m^3 = (100)^3\ cm^3 \Longrightarrow 1\ m^3 = (1\ 000\ 000)\ cm^3$

For example, converting 5.3 m^2 to cm^2 unit will be:

$$5.3\ m^2 \times \left(\frac{10000\ cm^2}{1\ m^2}\right) = 53000\ cm^2$$

1 cm = 10^{-2} m

$1\ cm^2 = (10^{-2})^2\ m^2 \Longrightarrow 1\ cm^2 = (10^{-4})\ m^2$

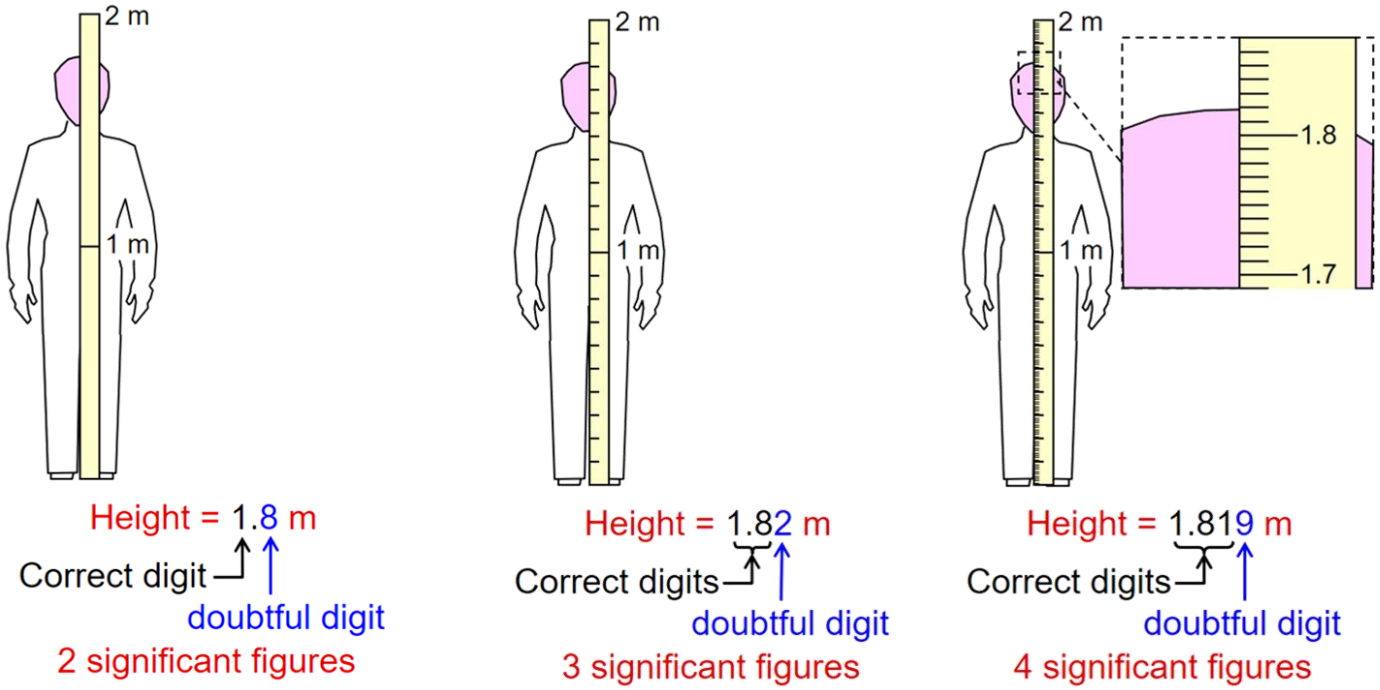
$1\ cm^3 = (10^{-2})^3\ m^3 \Longrightarrow 1\ cm^3 = (10^{-6})\ m^3$

For example, converting 5.3 cm^2 to m^2 unit will be:

$$5.3\ cm^2 \times \left(\frac{10^{-4}\ m^2}{1\ cm^2}\right) = 5.3 \times 10^{-4}\ m^2 = 0.00053\ m^2$$

Significant Figures and Decimal Places

Number of the significant figures depends on the instrument used



15.07 This number has four significant figures

10.68 The 8 is called the least significant figure

0.00064 This number has two significant figures

12.000 This number has five significant figures

RULES FOR SIGNIFICANT FIGURES

1. **All non-zero numbers ARE significant.**

The number 33.2 has THREE significant figures because all of the digits present are non-zero.

2. **Zeros between two non-zero digits ARE significant.**

2051 has FOUR significant figures. The zero is between a 2 and a 5.

3. **Leading zeros are NOT significant.**

They're nothing more than "place holders." The number 0.54 has only TWO significant figures. 0.0032 also has TWO significant figures. All of the zeros are leading.

4. **Trailing zeros to the right of the decimal ARE significant.**

There are FOUR significant figures in 92.00.

92.00 is different from 92: a scientist who measures 92.00 milliliters knows his value to the nearest 1/100th milliliter; meanwhile his colleague who measured 92 milliliters only knows his value to the nearest 1 milliliter. It's important to understand that "zero" does not mean "nothing." Zero denotes actual information, just like any other number. You cannot tag on zeros that aren't certain to belong there.

5. **Trailing zeros in a whole number with the decimal shown ARE significant.**

Placing a decimal at the end of a number is usually not done. By convention, however, this decimal indicates a significant zero. For example, "540." indicates that the trailing zero IS significant; there are THREE significant figures in this value.

6. **Trailing zeros in a whole number with no decimal shown are NOT significant.**

Writing just "540" indicates that the zero is NOT significant, and there are only TWO significant figures in this value.

7. **For a number in scientific notation: N x 10^x, all digits comprising N ARE significant by the first 6 rules; "10" and "x" are NOT significant.**

5.02 x 10^4 has THREE significant figures: "5.02." "10 and "4" are not significant.

Length

- In 1792, the meter (m) was selected to be the standard unit of length.
- The old definition of the meter: the meter, defined to be one ten-millionth of the distance from the north pole to the equator.
- Later, the meter was defined as the distance between two fine lines engraved near the ends of a platinum–iridium bar, the **standard meter bar**, which was kept at the International Bureau of Weights and Measures near Paris.

The standard meter

It is a bar of Platinum-Iridium kept at a constant temperature The meter is defined as the distance between the two scratch marks



- New definition of the meter: The meter is the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second.

Example: The micrometer (1 μm) is often called the *micron.* (a) How many microns make up 1 km? (b) What fraction of a centimeter equals 1 μm? (c) How many microns are in 1 yd?

Solution:

(a) Conversion factors:

1 km = 1×10^3 m and 1 m = 1×10^6 µm,

$\Longrightarrow$ 1 km = 10^3 m

Now we convert 10^3 m to µm:

$$10^3 \text{ m} \times \left(\frac{1 \times 106 \mu\text{m}}{1 \text{ m}}\right) = 10^9 \text{ µm}$$

(b) Conversion factors:

1 cm = 1×10^{-2} m and 1 µm = 1×10^{-6} m,

$\Longrightarrow$ 1 µm = 1×10^{-6} m

Now we convert 1×10^{-6} m to cm:

$$1 \times 10^{-6} \text{ m} \times \left(\frac{1 \text{ cm}}{1 \times 10^{-2} \text{ m}}\right) = 1 \times 10^{-4} \text{ cm}$$

(c) Conversion factors:

1 yd = 0.91 m and 1 m = 1×10^6 µ m,

$\Longrightarrow$ 1 yd = 0.91 m

Now we convert 0.91 m to µ m:

$$0.91 \text{ m} \times \left(\frac{1 \times 106 \mu\text{m}}{1 \text{ m}}\right) = 9.1 \times 10^5 \text{ µ m}$$

Time

- Any phenomenon that repeats itself is a possible time standard, for example, Earth's rotation, which determines the length of the day, has been used in this way for centuries
- However, the accuracy of measuring time this way is not good enough for science and engineering technology.
- To meet the need for a better time standard, atomic clocks have been developed
- One second is defined as the time taken by 9 192 631 770 oscillations of the light emitted by the cesium atom.
- Atomic clocks are so consistent and accurate.



Example: A plant can grow 3.7 m in 14 days. What was its growth rate in micrometers per second?

Solution:

Conversion factor:
$1\ m = 1 \times 10^6\ \mu m$ and $1\ day = 86400\ s$

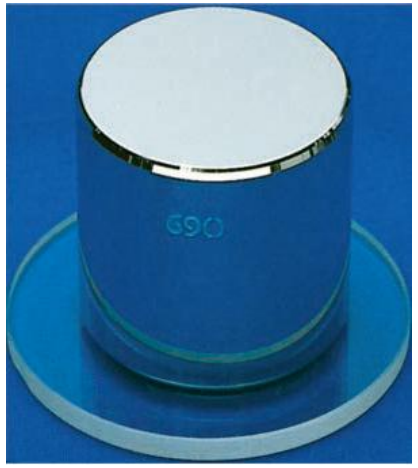
The growth rate in m/days = $\frac{3.7\ m}{14\ day} = 0.26\ m/day$

Now, convert 0.26 m/day to µm/s:

$$0.26\left(\frac{m}{day}\right) \times \left(\frac{1 \times 10^6\ \mu m}{1\ m}\right) \times \left(\frac{1\ day}{85400\ s}\right) = \left(\frac{0.26\ m \times 1 \times 10^6\ \mu m \times 1\ day}{1\ day \times 1\ m \times 86400\ s}\right) = 3\ \mu m/s$$

Mass

- Mass used to be measured by making a comparison to the “standard kilogram”



- The Standard Kilogram: The SI standard of mass is a cylinder of platinum and iridium that is kept at the International Bureau of Weights and Measures near Paris

- The masses of atoms can be compared with one another more precisely than they can be compared with the standard kilogram. For this reason, **the mass of carbon atom is used as a second mass standard**. By agreement,

mass of carbon atom = 12 atomic mass unit (u).

$1\ u = 1.66054 \times 10^{-27}\ kg$

- In 2019, the kilogram definition has changed (it is defined now based on some universal physics constants).

Example: Earth has a mass of 5.98×10^{24} kg. The average mass of the atoms that make up Earth is 40 u. How many atoms are there in Earth?

Solution:

Conversion factor:
$1\ \text{u} = 1.66054 \times 10^{-27}\ \text{kg}$

$$\Longrightarrow \text{mass of one atom} = 40\ \text{u} \times \left(\frac{1.66054 \times 10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 6.64 \times 10^{-26}\ \text{kg}$$

$$\text{Number of atoms in Earth} = \left(\frac{\text{mass of Earth}}{\text{mass of one atom}}\right) = \left(\frac{5.98 \times 1024\ \text{kg}}{6.64 \times 10\text{–}26\ \text{kg}}\right) = 9 \times 10^{49}\ \text{atoms}$$

Example: Assuming that water has a density of exactly 1 g/cm³, find the density of water in kg/m³.

Solution:

Conversion factor:

$1\ \text{g} = 1\times10^{-3}\ \text{kg}$ and $1\ \text{cm}^3 = 10^{-6}\ \text{m}^3$

$$1\left(\frac{\text{g}}{\text{cm}^3}\right)\times\left(\frac{1\times10^{-3}\ \text{kg}}{1\ \text{g}}\right)\times\left(\frac{1\ \text{cm}^3}{10^{-6}\ \text{m}^3}\right) = \left(\frac{10^{-3}}{10^{-6}}\right)\text{kg} = 10^3\ \text{kg/m}^3 = 1000\ \text{kg/m}^3$$

Example: A light-year is a measure of length (not a measure of time) equal to the distance that light travels in 1 year. Compute the conversion factor between light-years and meters.

Solution:

The conversion factor from years to second is

$$1y = 1y \times \frac{365.25d}{1y} \times \frac{24h}{1d} \times \frac{60min}{1h} \times \frac{60s}{1min} = 3.16 \times 10^7 s$$

The speed of light is $3.00 \times 10^8 m/s$.

Thus in 1 year, light travels a distance of

$$\left(3.00 \times 10^8\ m/s\right)\left(3.16 \times 10^7 s\right) = 9.48 \times 10^{15}\ m$$

So that

$$1\ ly\ = 9.48 \times 10^{15}\ m$$

Exercise 1 :

1. Express the following quantities using the prefixes and the unit abbreviations.

For example, 10000 meters = 10 km

a. 1 000 000 watts b. 0.002 gram c. 3 x 10^{-6} meters d. 30 000 seconds.

2. Write each of the following without using prefixes: (a) 40 µW (b) 4 ns (c) 3 MW (d) 25 km

3. Convert a speed of 10 km/h to mi/h.
4. Knowing that one gallon =231 in^3 and 1 in = 2.54 cm, then the Volume of 5 gallons of water equals to :

5. Given $1\ in = 2.54\ cm$ The volume of an object is $21\ cm^3$, then its volume in in^3 is:
6. Complete the following: (b) 100 yd =…… m (d) 60 mi/h =……. ft/s

7. Find the number of significant figures present in each of the following measurements 0.0040102, 1.52000, 6975400.

Chapter 3

Vectors

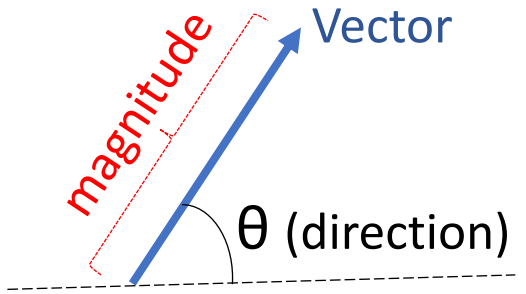
Vectors and Their Components

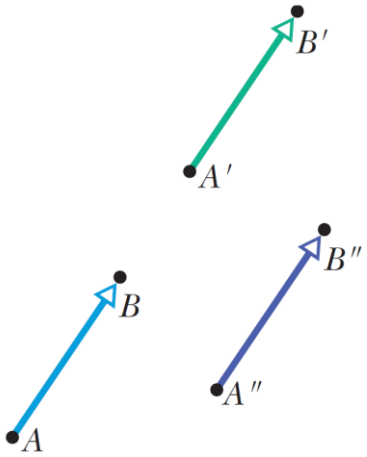
All physical quantities can be either

Vectors: have magnitude and direction
Examples: displacement, force, acceleration …

Scalars: have only magnitude (no direction)
Examples: distance, temperature, mass …

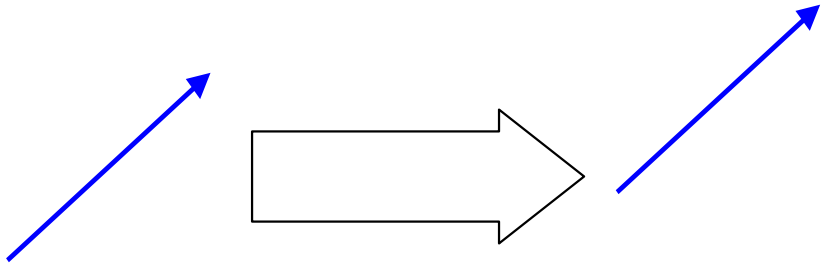
Vectors are represented graphically as arrows: The length of the arrow is proportional to the magnitude of the vector, and the direction (angel) of the arrow represents the direction of the vector.





Any vectors that have the same *magnitude* and the same *direction* are *equal vectors*.

In a diagram, you can move a vector to a new position if you don't change its length and direction.

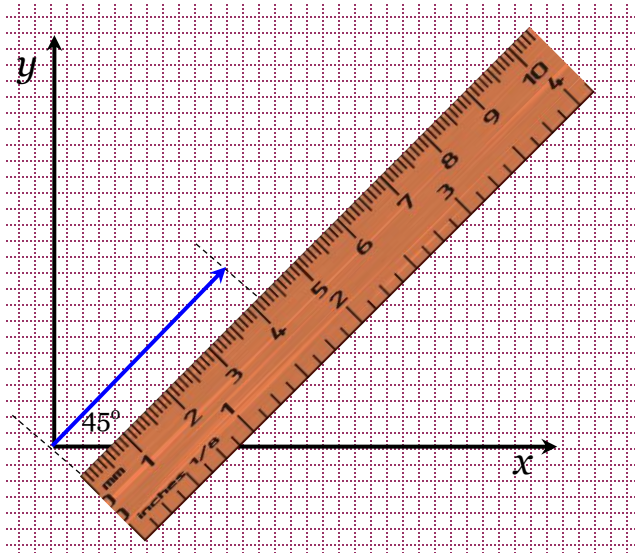


It is allowed to shift a vector to a position parallel to itself.

Vectors and Their Components

Representing the vector quantity graphically:

The vector is represented graphically as an arrow, whose length is proportional to its magnitude, and its slope represents the direction of the vector.



Symbols of the vector and its magnitudes

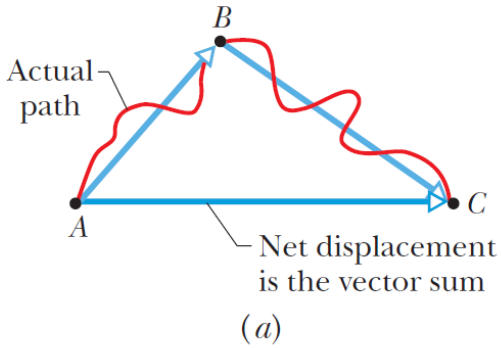
Symbols of the vector	$\boldsymbol{A}$	$\vec{A}$
	As a bold letter	As a letter with an arrow overhead
Symbols of the magnitude of the vector	A	$\left\vert\vec{A}\right\vert$ or $\vert\boldsymbol{A}\vert$
	As a simple letter	As a modulus of the vector

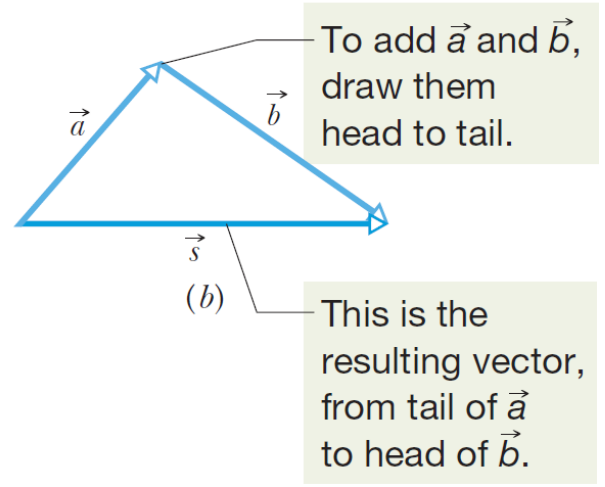
We can add vectors using two methods

Geometrically | By components

Adding Vectors Geometrically

- Suppose you moved from *A* to *B* as shown by the red line in the figure. Your displacement (which is a vector) is represented by the arrow for *A* to *B*. Next, you moved from *B* to *C* as shown. What is your net displacement? It is the vector for *A* to *C*.
- This is the way you can add vectors geometrically. *AB + BC = AC*
- We call *AC* the **vector sum** (or **resultant**) of the vectors *AB* and *BC*.
- We can represent the relation among the three vectors in the figure with the *vector equation:*

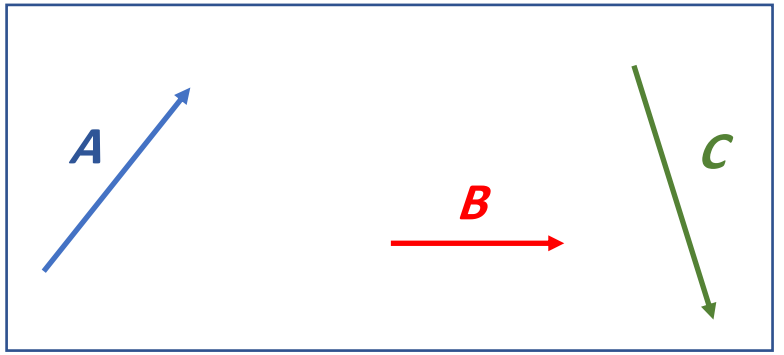




$$\vec{s} = \vec{a} + \vec{b}$$

This sum is not the usual algebraic sum (in vector sum, we add magnitudes and also direction!)

Adding Vectors Geometrically



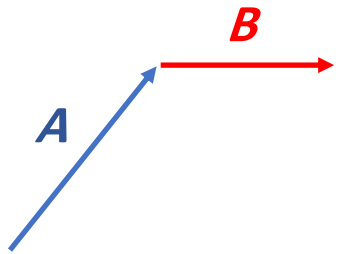
- The method of adding vectors geometrically is called head-to-tail method. It is done by the following steps:

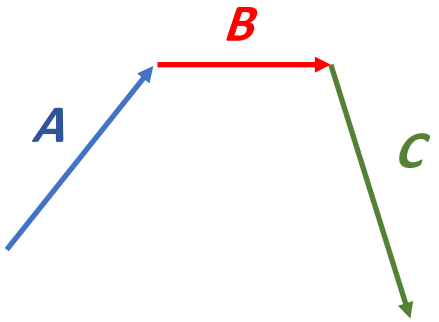
Assume you have 3 vectors ***A***, ***B*** and ***C*** that you need to add.

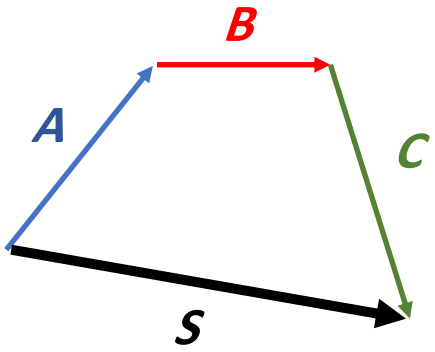
1) On paper, sketch vector ***A*** to some scale and at the proper angle.
2) Sketch vector ***B*** to the same scale, with its tail at the head of vector ***A***, again at the proper angle.

3) Repeat the same procedure for vector ***C***.

4) The vector sum ***S*** is the vector that extends from the tail of ***A*** to the head of ***C*** (the arrow represents the magnitude and direction of the resultant ***C***).





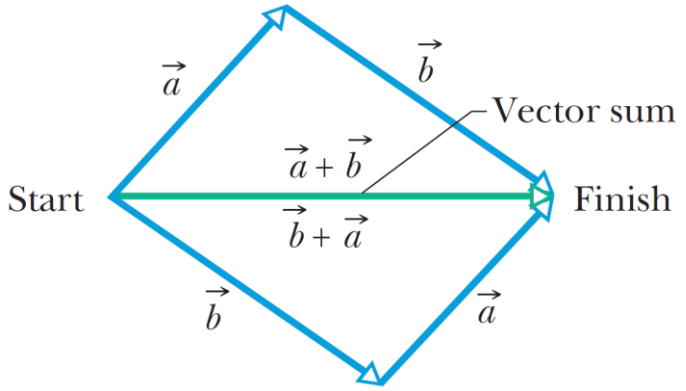


- This can be written as:

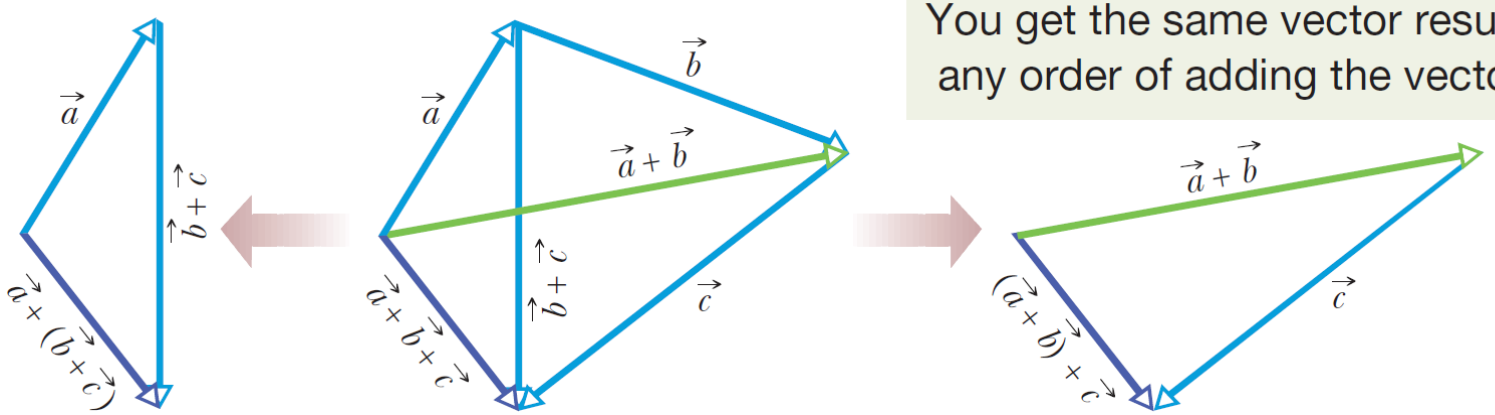
$$\boldsymbol{S} = \boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} \qquad \text{Or} \qquad \vec{S} = \vec{A} + \vec{B} + \vec{C}$$

Properties of Vector addition:

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ **(commutative law)**



- $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}\,)$ **(associative law)**



You get the same vector result for any order of adding the vectors.

Subtracting Vectors Geometrically

- The vector $-\vec{b}$ is a vector with the same magnitude as $\vec{b}$ but the opposite direction

This means:

$$\vec{b} + \left(-\vec{b}\right) = 0$$

- Thus, adding $-\vec{b}$ has the effect of subtracting $\vec{b}$. We use this property to define the difference between two vectors: let $\vec{d} = \vec{a} - \vec{b}$:

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \qquad \textbf{(vector subtraction)}$$

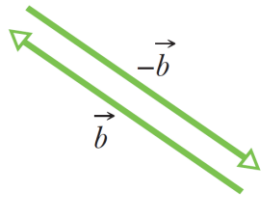


Figure 3-5 The vectors $\vec{b}$ and $-\vec{b}$ have the same magnitude and opposite directions.

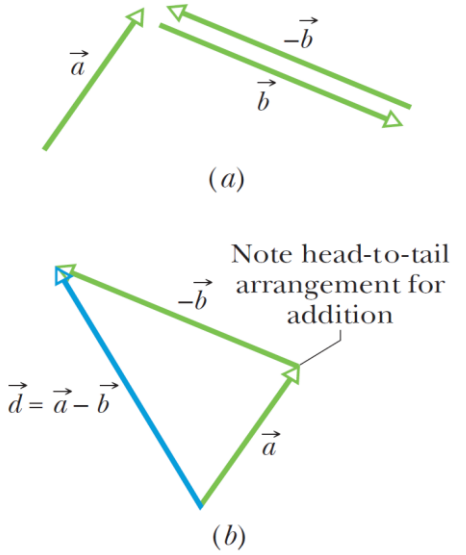
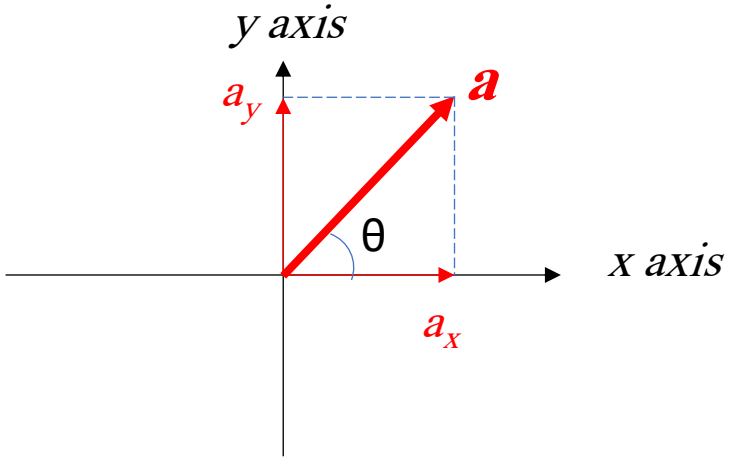
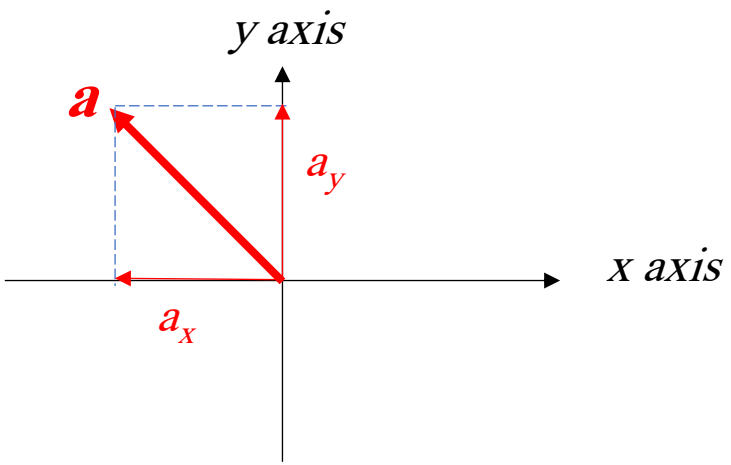


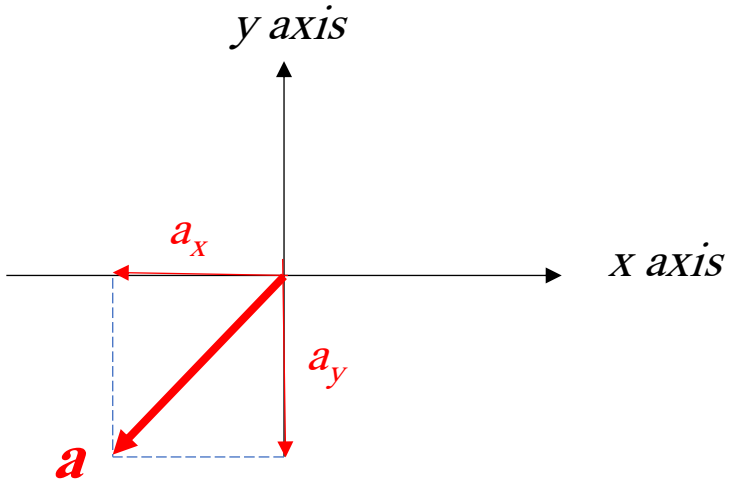
Figure 3-6 (*a*) Vectors $\vec{a}$, $\vec{b}$, and $-\vec{b}$. (*b*) To subtract vector $\vec{b}$ from vector $\vec{a}$, add vector $-\vec{b}$ to vector $\vec{a}$.

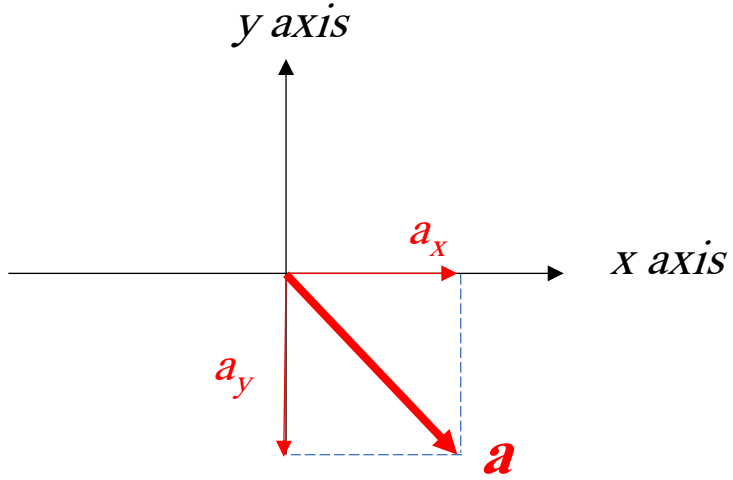
Components of Vectors

- We can ***resolve*** any vector to its components
- The components are the projections of the vector on each axis
- For example, the vector ***a*** has *x-axis* component a_x and *y-axis* component a_y
- The components can be positive or negative (based on the direction of the vector itself)





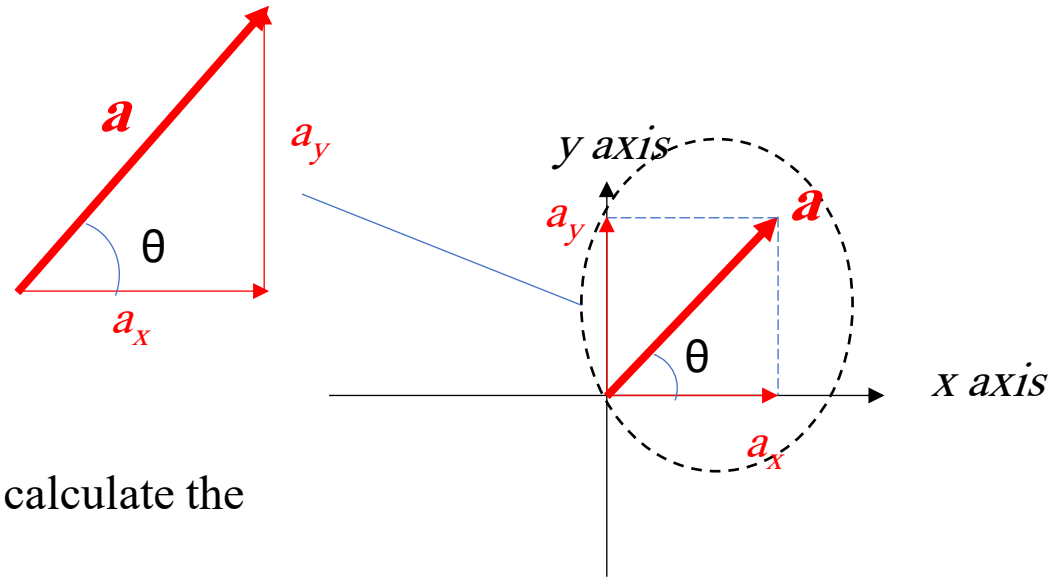




How can you convert a vector to its component?

$$cos\theta = \frac{a_x}{a} \rightarrow \boxed{a_x = a\ cos\theta}$$

$$sin\theta = \frac{a_y}{a} \rightarrow \boxed{a_y = a\ sin\theta}$$



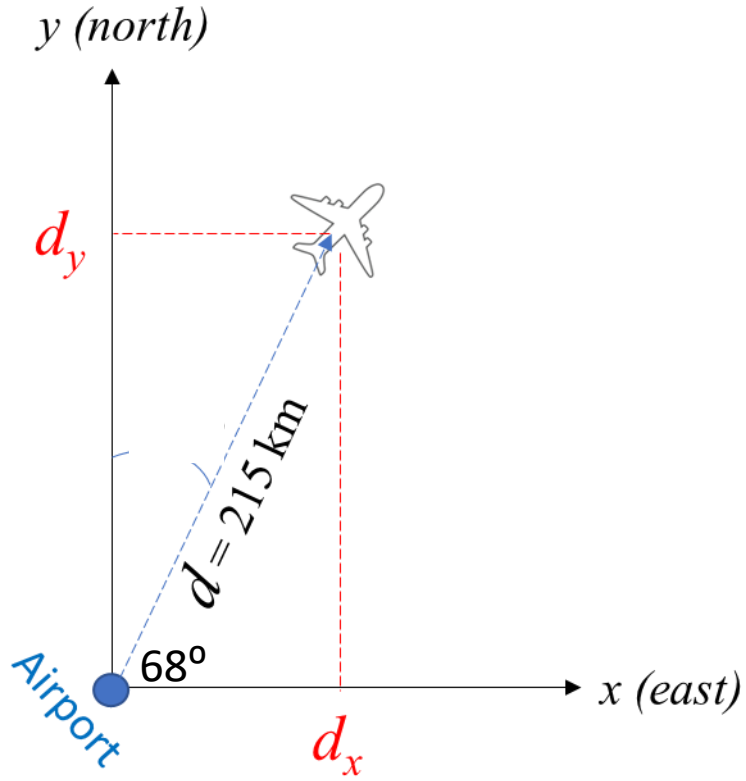
Also, if you know the components of any vector then you can calculate the magnitude and angle (direction) of the original vector:

$$\boxed{a = \sqrt{a_x^2 + a_y^2}}$$

$$\boxed{\theta = tan^{-1}\frac{a_y}{a_x}}$$

Note: these equations are correct only if the angle θ is made with the positive x-axis. If the angle is made with any other axis, you must recalculate these equations by the same method (see the next example)

Example: An airplane leaves an airport and moves for 215 km away, in a direction making an angle of 68° toward the east from due north. How far east and north is the airplane from the airport?



Solution:

We need to find the x-axis and y-axis components of the displacement d:

$d_x = d \cos\theta = 215 \times \cos(68°) = 81$ km

$d_y = d \sin\theta = 215 \times \sin(68°) = 200$ km

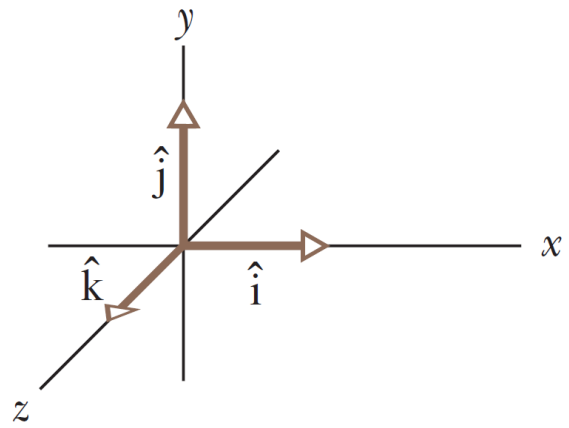
This means that the airplane is 81 km east and 200 km north of the airport.

UNIT VECTORS

Unit Vectors

- A **unit vector** is a vector that has a magnitude of exactly 1 and points in a particular direction.

The unit vectors point along axes.



- For each axis we have a particular unit vector:

x-axis has the unit vector $\hat{\imath}$
y-axis has the unit vector $\hat{\jmath}$
z-axis has the unit vector $\hat{k}$

- So, we can write the vector $\vec{a}$ as:

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

Adding Vectors by Components:

If we have two vectors $\vec{a}$ and $\vec{b}$: $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$, $\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath}$

and we need to calculate $\vec{r} = \vec{a} + \vec{b}$ we just add the components of each axis to each other:

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

$$+$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath}$$

$$\vec{r} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath}$$

For example,

if we have two vectors: $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ and $\vec{B} = -5\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

Then we can calculate $\vec{A} + \vec{B}$ as follows,

$$\vec{A} + \vec{B} = (3 - 5)\hat{\imath} + (4 + 3)\hat{\jmath} + (-2 + 2)\hat{k}$$

$$= -2\hat{\imath} + 7\hat{\jmath} + (0)\hat{k}$$

$$= -2\hat{\imath} + 7\hat{\jmath}$$

Multiplying Vectors

There are three ways in which vectors can be multiplied

Multiplying a Vector by a Scalar

Multiplying a vector $\vec{a}$ by a scalar s, we get a new vector s$\vec{a}$.
Its magnitude is the product of the magnitude of $\vec{a}$ and the value of s.

Multiplying a Vector by a Vector

Vector product (cross product), the result is a vector quantity

$$\vec{a} \times \vec{b} = vector\ quantity$$

Multiplying a Vector by a Vector

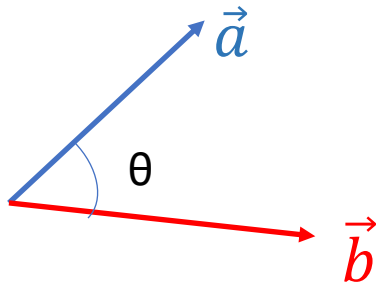
Scalar product (dot product), the result is a scalar quantity

$$\vec{a} \cdot \vec{b} = scalar\ quantity$$

The Scalar Product

There are 2 ways to calculate this type of product

If the vectors are expressed in magnitude-angle notation:



$$\vec{a} \cdot \vec{b} = ab\ cos\theta$$

- ∴ the result depends on the magnitude of *a* and the magnitude of *b* and the angle between them.
- the result can be positive or negative based on the values of the vectors, and the angle between them.

If the vectors are expressed in unit-vector notation:

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\vec{k}$$
$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\vec{k}$$

$$\vec{\imath} \cdot \vec{\imath} = \vec{\jmath} \cdot \vec{\jmath} = \vec{k} \cdot \vec{k} = 1$$

$$\vec{\imath} \cdot \vec{\jmath} = \vec{\jmath} \cdot \vec{k} = \vec{\imath} \cdot \vec{k} = 0$$

then,

$$\vec{a} \cdot \vec{b} = (a_x b_x) + (a_y b_y) + (a_z\ b_z)$$

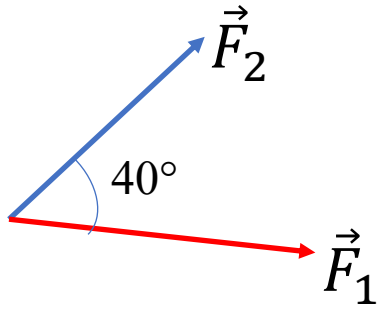
Note that the result has no unit vectors because it is a scalar quantity (the final result will be just a number)

The **commutative law** applies to the scalar product

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

<u>Example</u>: if you have two forces $\vec{F}_1$= 5 N and $\vec{F}_2$ = 3 N and the angle between them is 40°, find $\vec{F}_1 \cdot \vec{F}_2$

Solution:



$\vec{F}_1 \cdot \vec{F}_2 = 5 \times 3 \times \cos(40°) = 11.5$ N

<u>Example</u>: if you have two forces

$$\vec{F}_a = 2\hat{\imath} - 3\hat{\jmath} + \vec{k}$$

$$\vec{F}_b = 4\hat{\imath} + 5\hat{\jmath} + 10\vec{k}$$

find $\vec{F}_a \cdot \vec{F}_b$

Solution:

$\vec{F}_a \cdot \vec{F}_b = (2 \times 4) + (-3 \times 5) + (1 \times 10)$

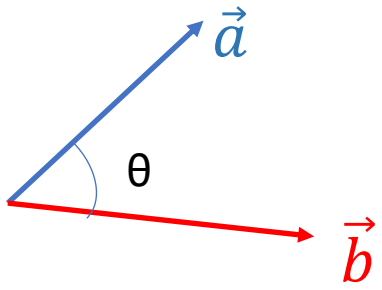
$= 8 - 15 + 10$

$= 3$ N

The Vector Product

The result of vector product of $\vec{a}$ and $\vec{b}$ is a third vector $\vec{c}$ whose magnitude can be calculated by two ways

If the vectors are expressed in magnitude-angle notation:



$$\vec{a} \times \vec{b} = ab\ sin\theta$$

- ∴ the result depends on the magnitude of *a* and the magnitude of *b* and the angle between them.
- the result can be positive or negative based on the values of the vectors, and the angle between them.

If the vectors are expressed in unit-vector notation:

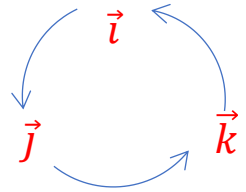
$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\vec{k}$$
$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\vec{k}$$

then,

$$\vec{a} \times \vec{b} = \begin{bmatrix} \vec{\imath} & j & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$

$$\vec{a} \times \vec{b} = [a_y b_z - b_y a_z]\,\hat{\imath} + [a_z b_x - b_z a_x]\,\hat{\jmath} + [a_x b_y - b_x a_y]\,\vec{k}$$

$$\vec{\imath} \cdot \vec{\imath} = \vec{\jmath} \cdot \vec{\jmath} = \vec{k} \cdot \vec{k} = 0$$



Note that the result has unit vectors because it is a vector quantity

The **commutative law** does not applies to the vector product

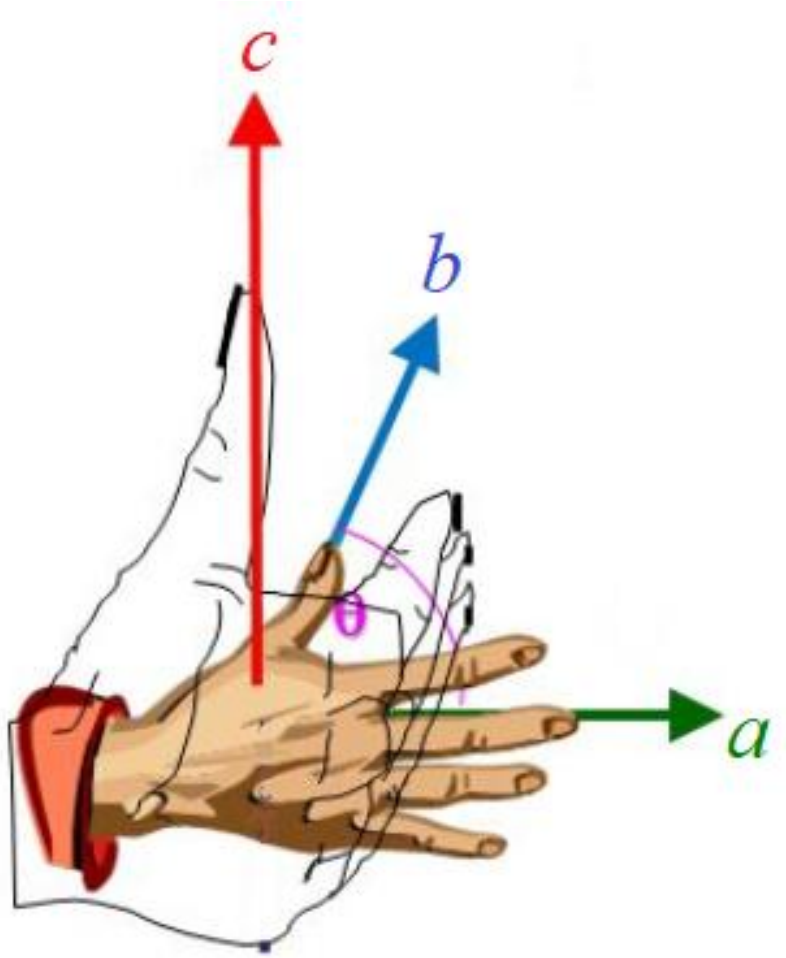
$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

To determine the **direction** of the result vector $\vec{c}$ we use **the right-hand-rule**·

1- point your right-hand fingers in the direction of $\vec{a}$.

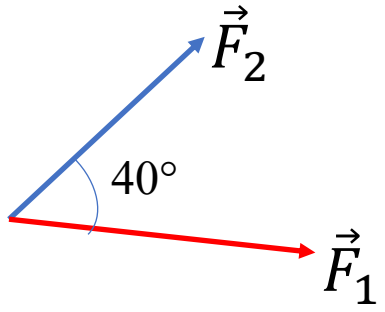
2- curl your fingers in the direction of $\vec{b}$.

3- the result vector $\vec{c}$ is in the direction of your thumb.



Example: if you have two forces $\vec{F}_1$= 5 N and $\vec{F}_2$ = 3 N and the angle between them is 40°, find $\vec{F}_1 \times \vec{F}_2$

Solution:



$\vec{F}_1 \times \vec{F}_2 = 5 \times 3 \times \sin(40°) = 12.6$ N

By using the right-hand-rule: the direction of the result is "out of the page"

Example: if you have two forces

$$\vec{F}_a = 2\hat{\imath} - 3\hat{\jmath} + \vec{k}$$

$$\vec{F}_b = 4\hat{\imath} + 5\hat{\jmath} + 10\vec{k}$$

find $\vec{F}_a \times \vec{F}_b$

Solution:

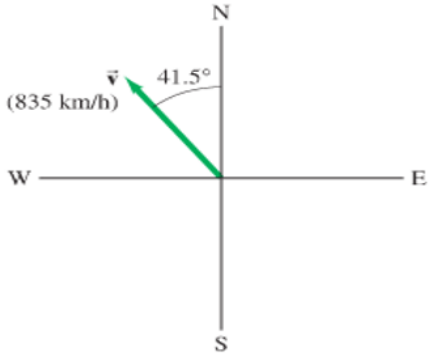
$$\vec{F}_a \times \vec{F}_b = [a_y b_z - b_y a_z]\,\hat{\imath} + [a_z b_x - b_z a_x]\,\hat{\jmath} + [a_x b_y - b_x a_y]\,\vec{k}$$

$$= [(-3\times10) - (5\times1)]\,\hat{\imath} + [(1\times4) - (10\times2)]\,\hat{\jmath} + [(2\times5) - (4\times-3)]\,\vec{k}$$

$$= -35\,\hat{\imath} - 16\,\hat{\jmath} + 22\,\vec{k}$$

Exercise 3:

1. An airplane is traveling 835 Km/h in a direction 41.5° west of north as shown in the figure Find:
a) The components of the velocity vector in the northerly and westerly directions
b) How far north and how far west has the plane travelled after 2.5 h?



2. The x component of a vector $\vec{A}$ is $-25\ m$ and the y component is $40\ m$.
(a) What is the magnitude of $\vec{A}$?
(b) What is the angle between the direction of $\vec{A}$ and the positive direction of x?

3. What is the angle Ø between $\vec{a} = 3\hat{\imath} - 4\hat{\jmath}$ and $\vec{b} = -2\hat{\imath} + 3\hat{k}$? what is $\vec{c} = \vec{a} \times \vec{b}$

4. In unit vector notation, what is the sum $\vec{a} + \vec{b}$ if $\vec{a} = 4\hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = -13\hat{\imath} + 7\hat{\jmath}$?
What are the magnitude and direction of $\vec{a} + \vec{b}$?

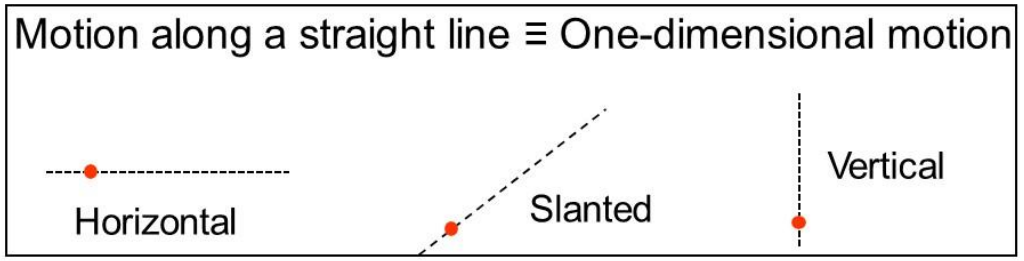
5. Two vectors are presented as $\vec{a} = 3\hat{\imath} + 5\hat{\jmath}$ and $\vec{b} = 2\hat{\imath} + 4\hat{\jmath}$ Find:
(a) $\vec{a} \times \vec{b}$
(b) $\vec{a}.\vec{b}$
(c) $\left(\vec{a} + \vec{b}\right).\vec{b}$

Chapter 2

Motion Along a Straight Line

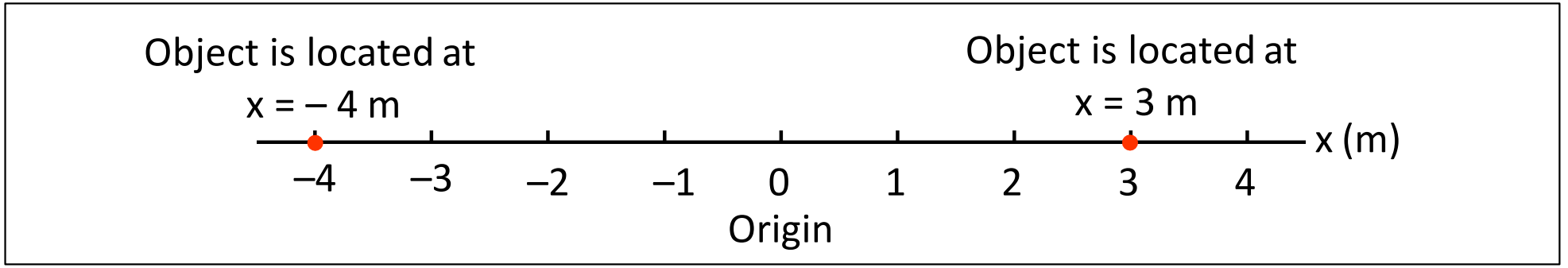
Motion:

- In this chapter we will study the motion of objects only in a straight line.
- We will not study the force causing this motion (this part will come later)
- Motion in a straight line can be vertical, horizontal, or slanted, but it must be straight.



Position and Displacement

- To locate an object means to find its position relative to some reference point, often the **origin** (or zero point) of an axis such as the *x* axis.
- Position can be on the **positive direction** or on the **negative direction** of the axis.



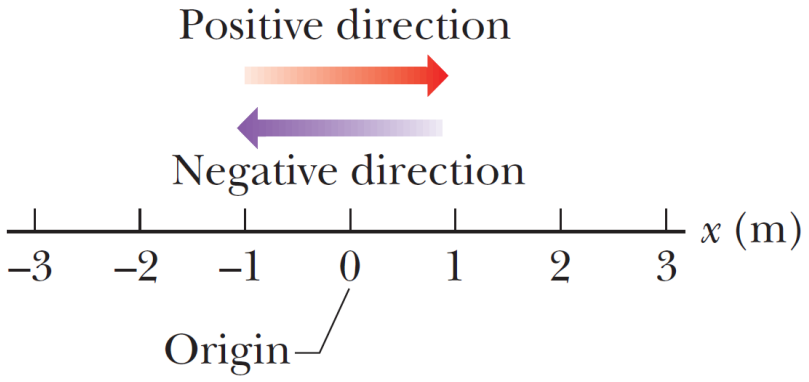
Position and Displacement

- A change from position x_1 to position x_2 is called a **displacement** Δx, where

$$\Delta x = x_2 - x_1$$

- For example, if the particle moves from $x_1 = 5$ m to $x_2 = 12$ m, then the displacement is $\Delta x = (12\text{ m}) - (5\text{ m}) = +7$ m. The positive result indicates that the motion is in the positive direction.

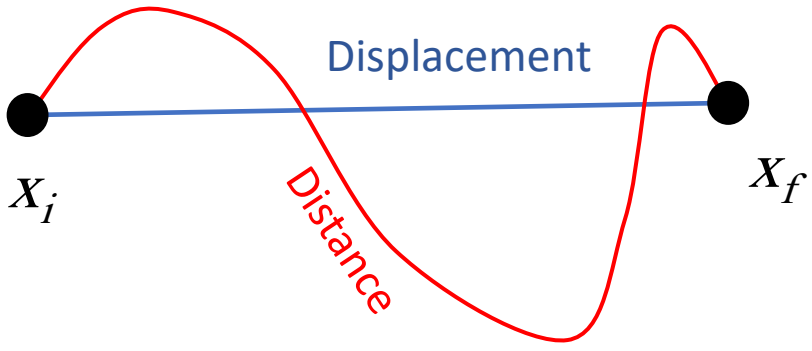
- If the particle moves from $x_1 = 5$ m to $x_2 = 1$ m, then $\Delta x = (1\text{ m}) - (5\text{ m}) = -4$ m. The negative result indicates that the motion is in the negative direction.



Position and Displacement

- Displacement is not always equal to the total distance covered in the trip
- Displacement is the net change in position between original and final positions
- Displacement is a vector: it is described by a *magnitude* and a *direction*
- Distance is a scalar (it has no direction)

- If you move between initial position x_i to a final position x_f as shown, then displacement ≠ distance



Displacement	Distance
Net change between initial position x_i to a final position x_f	Total trip from initial position x_i to a final position x_f
Does not depend on the path of motion	Depends on the path of motion
Vector	Scalar
SI unit: m	SI unit: m

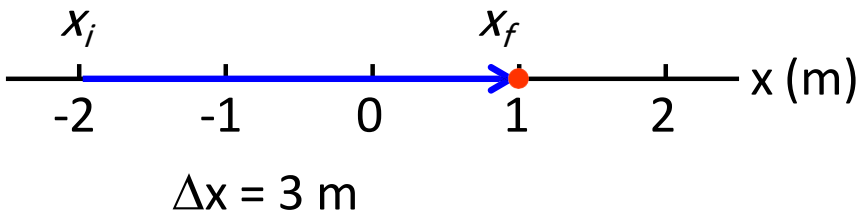
Position and Displacement

∵ Displacement is a vector

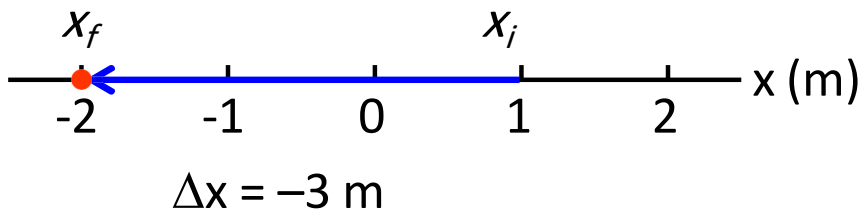
∴ To determine the displacement of an object, you need to specify

1- Magnitude (The distance between the initial and final positions)

2- Direction (Negative or positive sing)



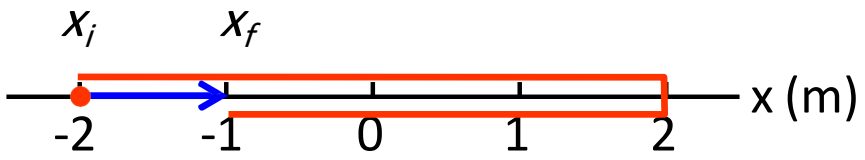
Displacement $\Delta x = 3$ m means the object position has changed by 3 m in the positive direction.



Displacement $\Delta x = -3$ m means the object position has changed by 3 m in the negative direction.

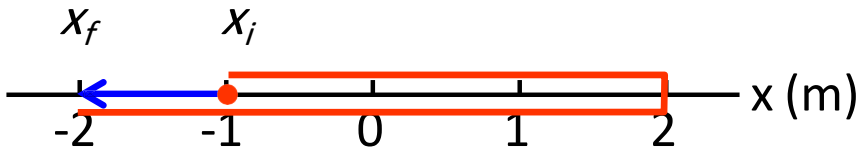
Position and Displacement

Here you can see the difference between displacement and distance on the *x*-axis:



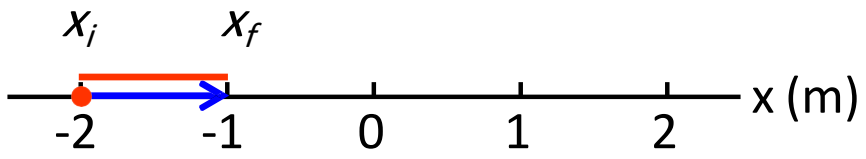
$\Delta x = 1$ m.

Total distance = 7 m.



$\Delta x = -1$ m.

Total distance = 7 m.



$\Delta x = 1$ m.

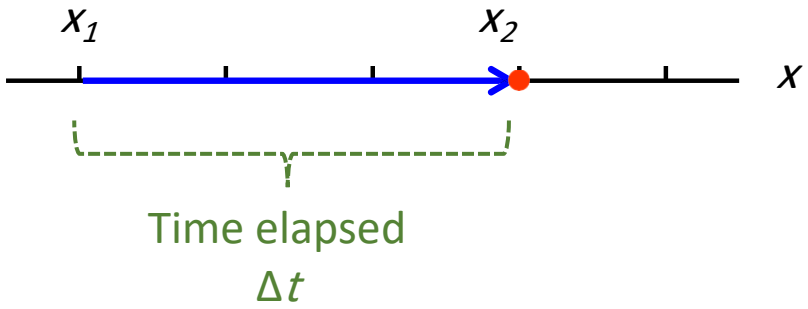
Total distance = 1 m.

Average Velocity and Average Speed

To measure "how fast" an object is moving, we use two quantities:

1) **average velocity** v_{avg}
2) **average speed** s_{avg}

Suppose an object is at position x_1 at time t_1, and then moves and reaches position x_2 at time t_2.



This means that the displacement Δx occurs during a particular time interval Δt

The **average velocity** V_{avg} is defined as:

$$average\ velcoity = \frac{displacement}{time} = \frac{final\ postion\ - initial\ position}{final\ time\ - initial\ time}$$

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

Average Velocity and Average Speed

- Because displacement is a vector ⟹ velocity is a vector (velocity always has direction)
- Average velocity v_{avg} always has the same sign as the displacement Δx:
  positive displacement ⟹ positive velocity
  negative displacement ⟹ negative velocity
- The SI unit of velocity is m/s

- **Average speed s_{avg}** is the second way of describing "how fast" an object moves.
- The average speed involves the total ***distance*** covered during the motion:

$$average\ speed = \frac{total\ distance}{time} = \frac{total\ distance}{final\ time\ - initial\ time}$$

$$s_{avg} = \frac{total\ distance}{\Delta t}$$

- Because distance is scalar, speed is scalar (has no direction).
- Average speed S_{avg} is always positive.
- The SI unit of speed is m/s

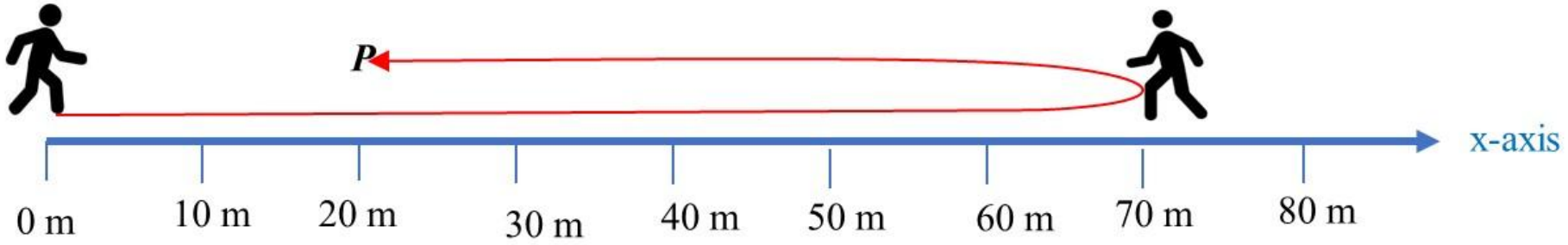
Average Velocity and Average Speed

Comparison between velocity and speed:

Velocity	Speed
$= \frac{\text{displacement}}{\text{time}}$	$= \frac{\text{distance}}{\text{time}}$
Does not depend on the path of motion	Depends on the path of motion
Vector	Scalar
SI unit: m/s	SI unit: m/s

- Average speed is not always equal to the magnitude of average velocity.
- Both v_{avg} and s_{avg} are measured over a time interval Δt (not for a specific instant during the motion), thus they are called “average”

Example: a man walked as shown in the figure, then stopped at the point *P*. The total time that he takes during his walk was 250 s. calculate the distance, displacement, average speed, and average velocity.



Solution:

Distance = 70 + 50 = 120 m

Displacement = +20 m

The + sign means that the displacement and the velocity are in the positive direction of x-axis

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{120}{250} = 0.48 \text{ m/s}$$

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{+20}{250} = +0.08 \text{ m/s}$$

Instantaneous Velocity and Speed

- You have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval Δt.

- However, we can also describe how fast a particle is moving at a given instant—its **instantaneous velocity** (or simply **velocity**) v.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\boxed{v = \frac{dx}{dt}}$$

- Velocity (v) is the derivative of position (x) with respect to time (t)

- v is a vector (it has direction)

- To calculate the instantaneous velocity at a time = t_o:
  1) Calculate the first derivative of the position equation with respect to time
  2) Substitute the value of t_o at the result of derivation

<u>Example</u>: If a particle's position is given by $x = 4 - 12t + 3t^2 - 5t^3$ (where t is in seconds and x is in meters), what is its velocity at $t = 2$ s?

<u>Solution</u>:

$$v = \frac{dx}{dt} = \frac{d}{dt}(4 - 12t + 3t^2 - 5t^3)$$

$$= 0 - 12 + 6t - 15t^2$$

At $t = 2$ s,

$$v = 0 - 12 + 6(2) - 15(2)^2$$

$$= -12 + 12 - 15(4) = -60 \text{ m/s}$$

This result means that at the instant $t = 2$ s, the particle was moving with a velocity = 60 m/s in the negative direction.

- Using the instantaneous velocity, we can define the **instantaneous speed** or simply **speed**

- **Speed** is the magnitude of velocity; that is, speed is velocity but without any direction.

- For example, a velocity of +5 m/s and another velocity of −5 m/s both have an associated speed of 5 m/s.

- Speed is scalar (it has no direction), and it is always positive.

- Note: speed and average speed can be different.

Acceleration

- When a particle's velocity changes, the particle is said to have **acceleration**
- Suppose an object is has a velocity v_1 at time t_1, and then changes its velocity to v_2 at time t_2, then, the **average acceleration** a_{avg} is defined as

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- Also, there is **instantaneous acceleration** or simply **acceleration** *a*, which is defined as:

$$a = \frac{dv}{dt}$$

Or,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Acceleration is the first derivative of v with respect to t

OR

Acceleration is the second derivative of x with respect to t

- To calculate the instantaneous acceleration at a time = t_o:

  1) Calculate the first derivative of the velocity equation with respect to time

  OR

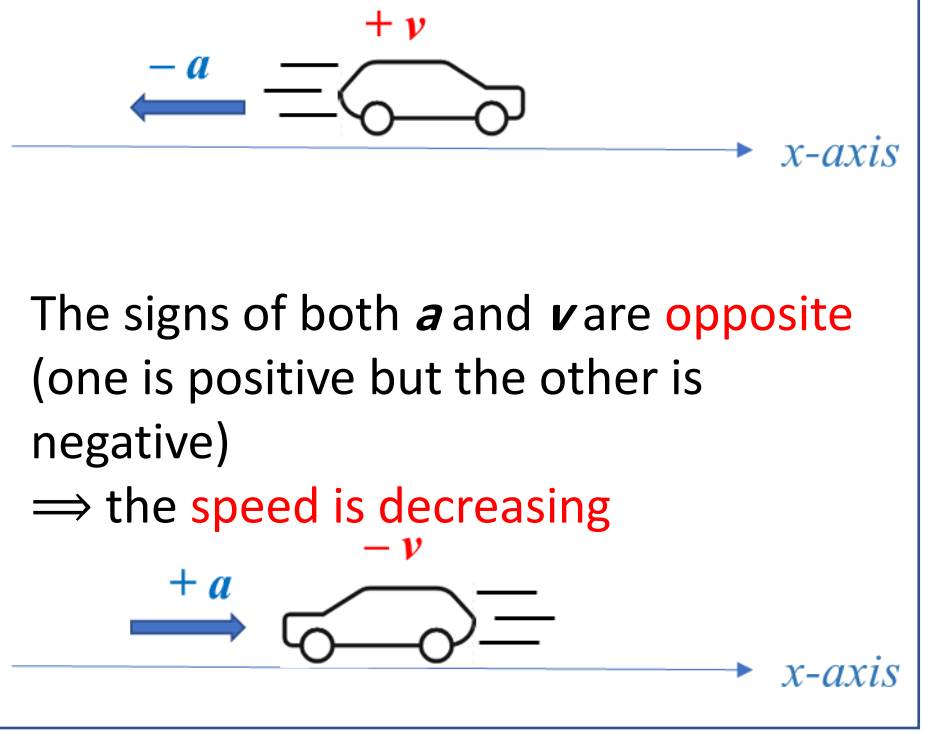
  calculate the second derivative of the position equation with respect to time

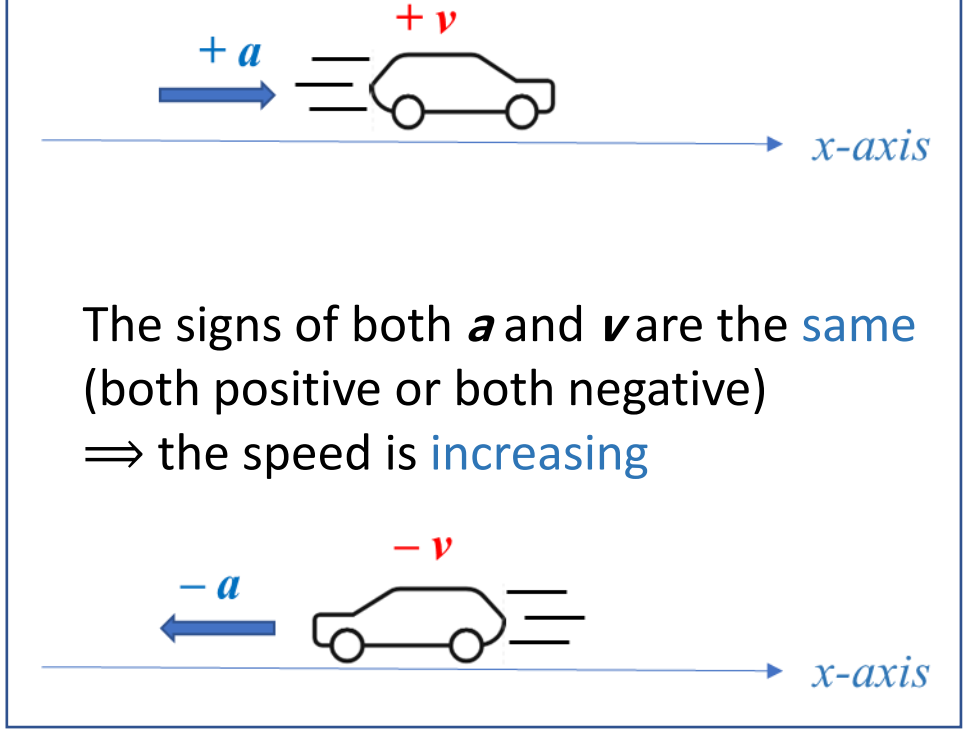
  2) Substitute the value of t_o at the result of derivation

- Acceleration is vector (it has direction)

- The SI unit of acceleration is m/s^2

- Acceleration can be **positive** or **negative**.
- If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.





<u>Example</u>: If a particle's position is given by $x = 4 - 12t + 3t^2 - 5t^3$ (where t is in seconds and x is in meters), what is its acceleration at $t = 2$ s?

Solution:
We can derive the velocity v from the last example (it is the same as taking the *second derivative* of the equation given in the question),

We had: $v = \frac{dx}{dt} = \frac{d}{dt}(4 - 12t + 3t^2 - 5t^3)$

$\therefore\ v = 12 + 6t - 15\,t^2$

$\Longrightarrow\ a = \frac{dv}{dt} = \frac{d}{dt}(12 + 6t - 15\,t^2)$

$\therefore a = 6 - 30\ t$

At $t = 2$ s,

$a = 6 - 30\ (2) \ = \ 6 - 60 = -54$ m/s^2

Constant Acceleration: A Special Case



- When an object is moving with a *constant acceleration* (this means its velocity is changing but not its acceleration), in this case:

  average acceleration = instantaneous acceleration

  $a_{avg} = a$

- We can describe this motion by a set of **equations** (*equations of motion with constant acceleration*)

$$\begin{cases} v = v_0 + at \\ x - x_0 = v_0 t + \frac{1}{2}at^2 \\ v^2 = v_0^2 + 2a(x - x_0) \\ x - x_0 = \frac{1}{2}(v_0 + v)t \\ x - x_0 = vt - \frac{1}{2}at^2 \end{cases}$$

In these equations:

v_o is the initial velocity
v is the final velocity
a is acceleration
t is time
x_o is the initial position
x is the final position

<u>Example:</u> A car starts moving from rest with a constant acceleration of 2 m/s². how long does the car take to travel a distance of 200 m? b) what is the car's final velocity at this point?

Solution:

$\because$ the car starts moving from rest $\Longrightarrow v_o = 0$ m/s
$a = 2$ m/s²
$x = 200$ m
$t = ?$

a) We can use the equation:

$$x = v_o t + \frac{1}{2} a t^2$$

$$200 = 0 \times t + \frac{1}{2} \times 2 \times t^2$$

$$200 = 0 + t^2$$

$$t = \sqrt{200} = 14.14 \text{ s}$$

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x - x_0 = \tfrac{1}{2}(v_0 + v)t$$
$$x - x_0 = vt - \tfrac{1}{2} a t^2$$

b) Now we know:

$v_o = 0$ m/s
$a = 2$ m/s²
$x = 200$ m
$t = 14.14$ s

To calculate v we can use the equation:

$$v = v_o + at$$

$$v = 0 + 2 \times 14.14$$

$v = 28.28$ m/s

Example: A car moving with a uniform velocity of 15 m/s is brought to rest in travelling a distance of 5 m. Calculate the deceleration produced by brakes?

Solution:

$$v = v_0 + at$$
$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x - x_0 = \tfrac{1}{2}(v_0 + v)t$$
$$x - x_0 = vt - \tfrac{1}{2}at^2$$

$\because$ the car is brought to rest $\Longrightarrow$ $v = 0$ m/s

$v_o = 15$ m/s

$x = 5$ m

$a = ?$

$$v^2 = v_o^2 + 2ax$$

$$0 = 15^2 + 2 \times a \times 5$$

$$0 = 225 + 10\,a$$

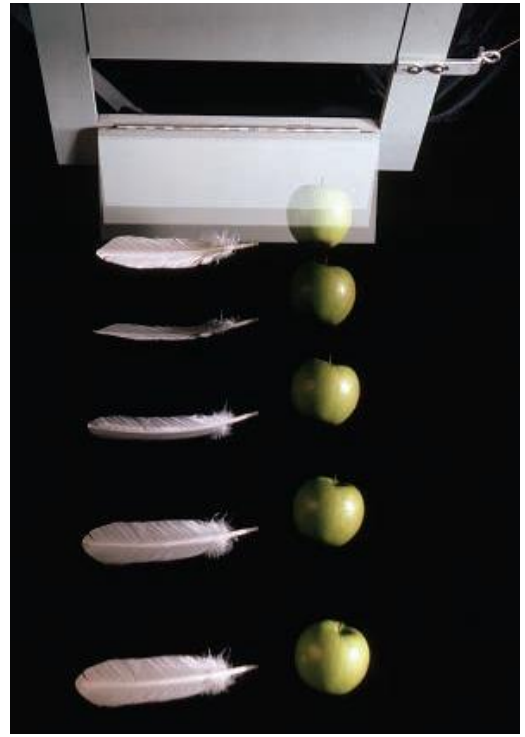
$$10\,a = -225$$

$$a = -\frac{225}{10}$$

$a = -22.5$ m/s^2

The minus sign means that the can was slowing down (decelerating) as mentioned in the question.

Free-Fall Acceleration

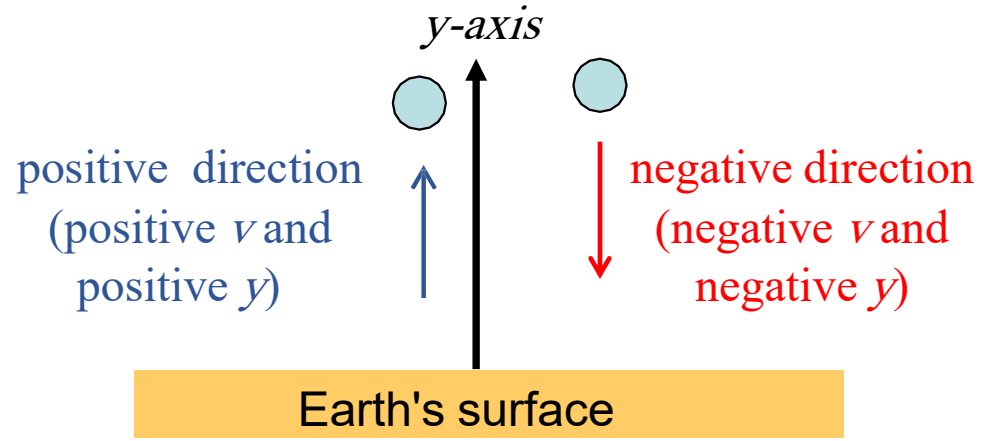


- If you eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate.
- That rate is called the **free-fall acceleration,** and its magnitude is *constant* and represented by g
- $g = 9.8\ \text{m/s}^2$ the g value (acceleration) is constant

∵ we can use for free-falling motion the same equations of motion that we just learned, only with the following changes:

(1) The directions of motion is along a vertical y axis instead of x axis, with the positive direction of y upward.

(2) The free-fall acceleration is always negative because it is always downward on the y-axis, toward Earth's center and so it has the value $-g$ in the equations



When an object is moving under the affect of Earth's gravity, we deal with it as a "*falling*" object (in moving both *upward* and also *downward*). The only difference between "falling" upward and falling downward is the signs of the velocity *v* and the distance *y*.

The equation of motion for a free-falling object are:

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t$$

$$x - x_0 = vt - \tfrac{1}{2}at^2$$



Equation of free falling
$v = v_o - gt$
$y - y_o = v_o t - \frac{1}{2}gt^2$
$v^2 = v_o^2 - 2g(y - y_o)$
$y - y_o = \frac{1}{2}(v_o + v)t$
$y - y_o = vt + \frac{1}{2}gt^2$

Remember: when you substitute *g* in the equations, you use $g = 9.8\ m/s^2$ (do not put $g = -9.8\ m/s^2$)

Example: A man throws a ball up along a *y* axis, with an initial speed of 12 m/s.

a) How long does the ball take to reach its maximum height?

b) What is the ball's maximum height above its release point?

Solution:

a) At the maximum height, the ball *stops* (for a short period of time) before falling back again

$\Longrightarrow v = 0$ m/s

We have:

$v_o = 12$ m/s

$g = 9.8$ m/s^2

$$v = v_o - gt$$

$$0 = 12 - 9.8\,t$$

$$-9.8\,t = -12$$

$$t = \frac{-12}{-9.8} = 1.22 \text{ s}$$

b) We can use the equation:

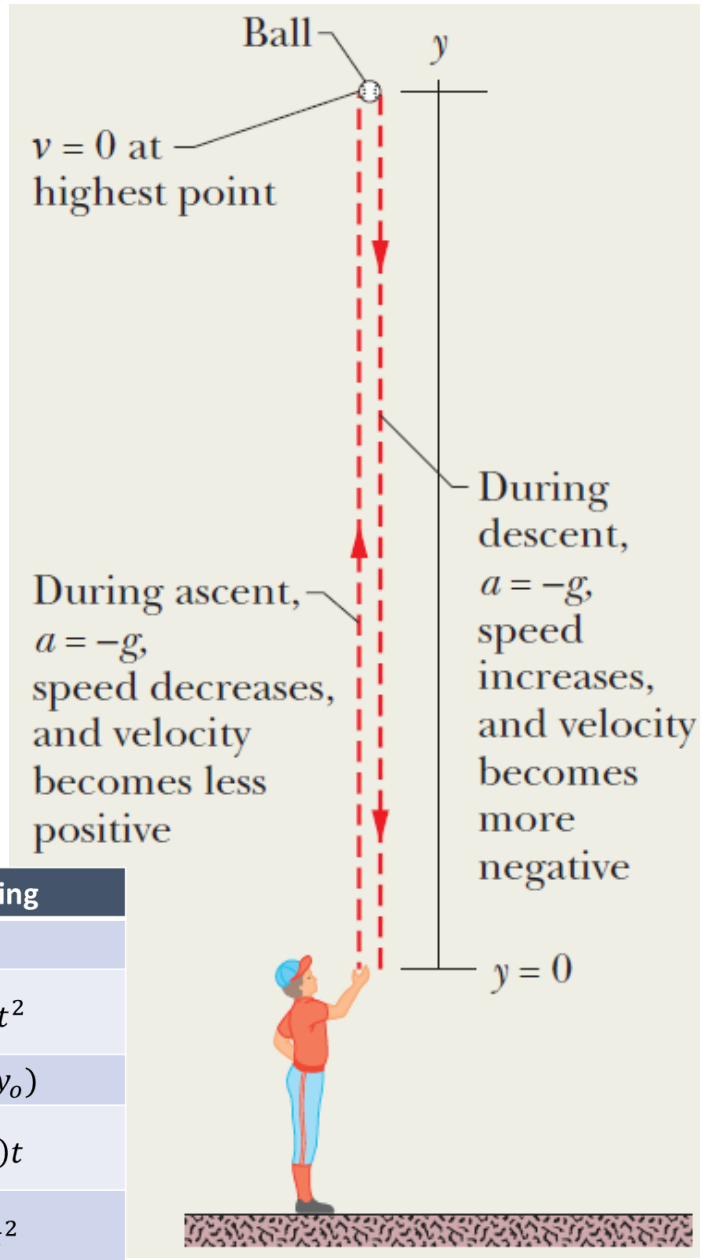
$$v^2 = v_o^2 - 2gy$$

$$0^2 = 12^2 - 2 \times 9.8 \times y$$

$$0 = 144 - 19.6\,y$$

$$y = \frac{-144}{-19.6}$$

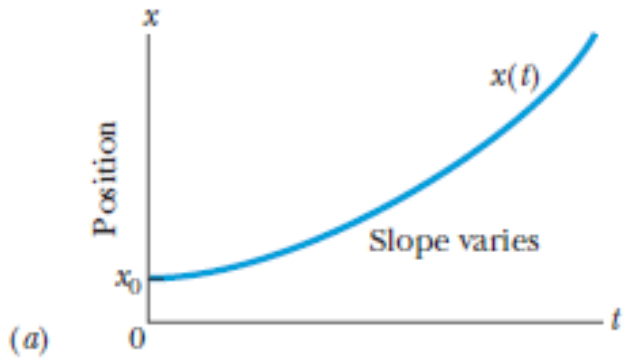
$y = 7.3$ m



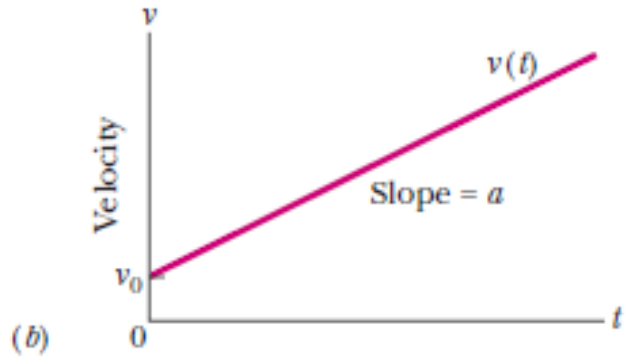
Equation of free falling
$v = v_o - gt$
$y - y_o = v_o t - \frac{1}{2}gt^2$
$v^2 = v_o^2 - 2g(y - y_o)$
$y - y_o = \frac{1}{2}(v_o + v)t$
$y - y_o = vt + \frac{1}{2}gt^2$

Graphical representation of Motion Analysis

In many types of motion, we can represent the motion graphically. In this case, graphs of position, velocity and acceleration with time could be roughly described as shown.



Position-Time graph shows that the object is moving while time elapses



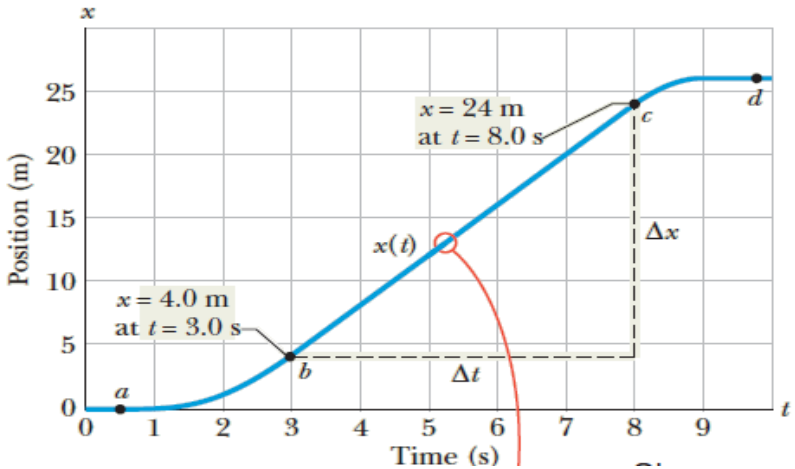
The velocity-Time graph shows that the velocity increases constantly (straight line) with time



The acceleration-Time graph shows that the velocity increases constantly with time, hence the acceleration is constant

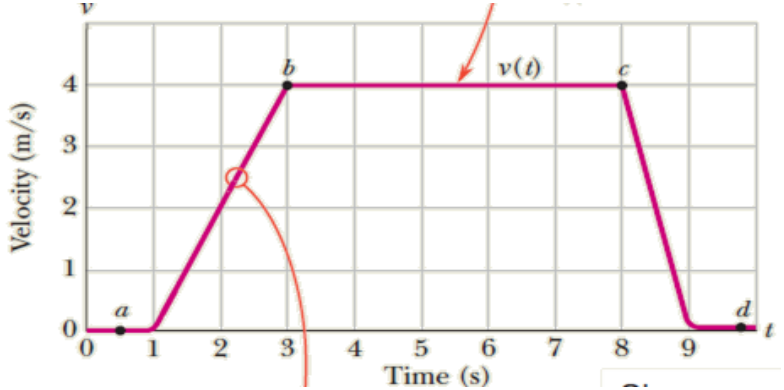
Graphical representation of Motion Analysis

This example shows a journey of a bus traveling between two stops.



Position-Time Graph Analysis

1. Bus is stopping for one second
2. It then starts moving, and its position changes with time.
3. At “b”, the bus travels equal distances in equal time.
4. At “c”, the bus prepares to stop, before stopping completely at “d”

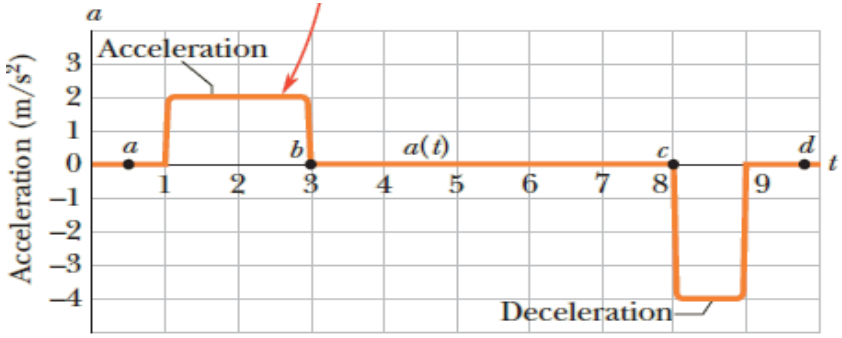


Velocity-Time Graph Analysis

1. the bus has a velocity=0 for one second
2. It then starts moving, and its velocity increases with time.
3. At “b”, to “c”, the bus travels with a constant velocity.
4. At “c”, the bus is reducing velocity, before stopping completely at “d”

Acceleration-Time Graph Analysis

1. the bus has an acceleration =0 for one second
2. In the time between t = 1 and 3, the bus has +ve acceleration.
3. At “b”, to “c”, the bus has an acceleration = 0.
4. At “c”, the bus has negative acceleration
5. the bus has “zero” acceleration at “d”

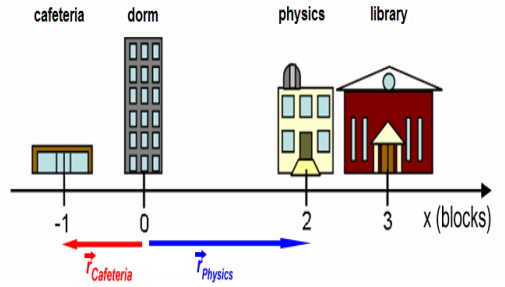


Exercise 2 :

1. The position of an object is given as a function of time by : $\mathbf{x = 15 + 4t - 2t^3}$, where x is in kilometres and t is in hours. According to this equation,

a) what is the object's starting point?
b) what is the average acceleration during the interval from t = 1.0 h to t = 2.0 h?
c) what is the acceleration at t=2.0 h?

2. In the morning, A student walked from the dorm to the library then to his class in the physics building in 4 minutes. **A.** Calculate the distance, displacement ,average speed, and average velocity. **B.** In the afternoon, he left the dorm to the cafeteria and then came back. Calculate the distance and displacement.



3. Raindrop fall 1000 m from a cloud to the ground.

a. How fast would the drop be moving when they struck the ground?

b. How long would it take?

4. An electron has a constant acceleration of 3.2 m/s^2. At a certain instant, its velocity is 9.6 m/s. What is its velocity after 2.5 s?

5. On a dry road, a car with good tires may be able to brake with a constant deceleration of 4.92 m/s^2

a. How long does such a car, initially traveling at 24.6 m/s, take to stop?

b. How far does it travel in this time?

Chapter 4

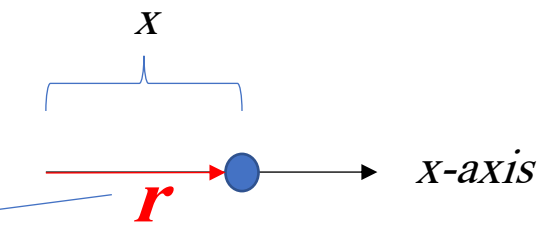
Motion in Two and Three Dimensions

Position and Displacement

The position of an object is described by the **position vector** $\vec{r}$

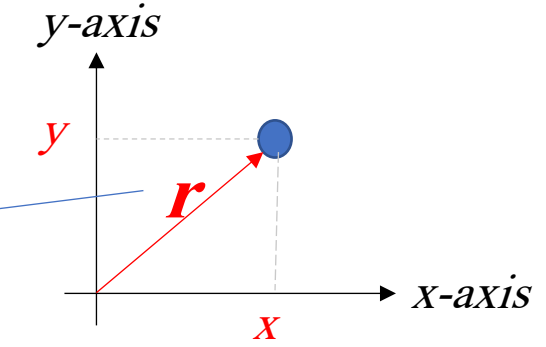
In one dimension (*x-axis* for example):

$$\vec{r} = x\hat{\imath}$$



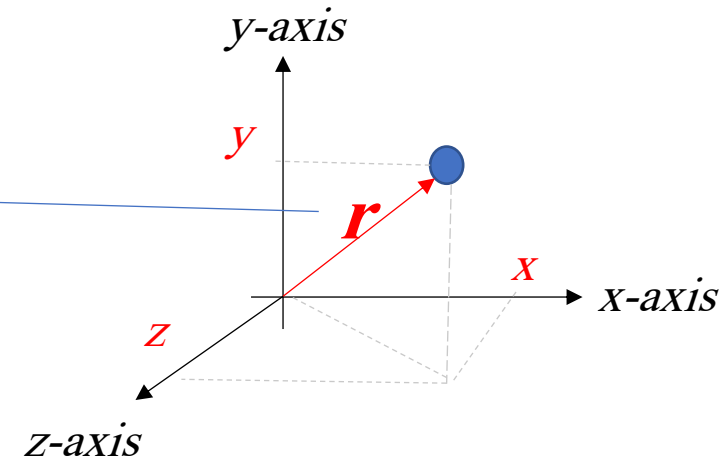
In two dimensions (*xy*):

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$



In three dimensions (*xyz*):

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

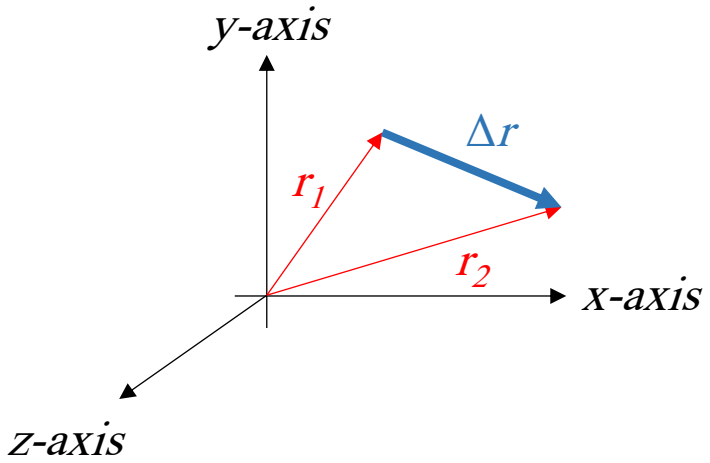


x, *y* and *z* are the coordinates on the *x-axis, y-axis*, and *z-axis*, respectively.

Position and Displacement

If the object moves and changes its position from an initial position $\overrightarrow{r_1}$ to a final position $\overrightarrow{r_2}$, where,

$\overrightarrow{r_1} = x_1\hat{\imath} + y_1\,\hat{\jmath} + z_1\,\hat{k}$ and $\overrightarrow{r_2} = x_2\hat{\imath} + y_2\,\hat{\jmath} + z_2\,\hat{k}$



then its displacement will be $\Delta\vec{r}$, where,

$$\Delta\vec{r} = \overrightarrow{r_2} - \overrightarrow{r_1}$$

$$= \left(x_2\hat{\imath} + y_2\,\hat{\jmath} + z_2\,\hat{k}\right) - \left(x_1\,\hat{\imath} + y_1\,\hat{\jmath} + z_1\,\hat{k}\right)$$

$$= (x_2 - x_1)\,\hat{\imath} + (y_2 - y_1)\,\hat{\jmath} + (z_2 - z_1)\,\hat{k}$$

Or,

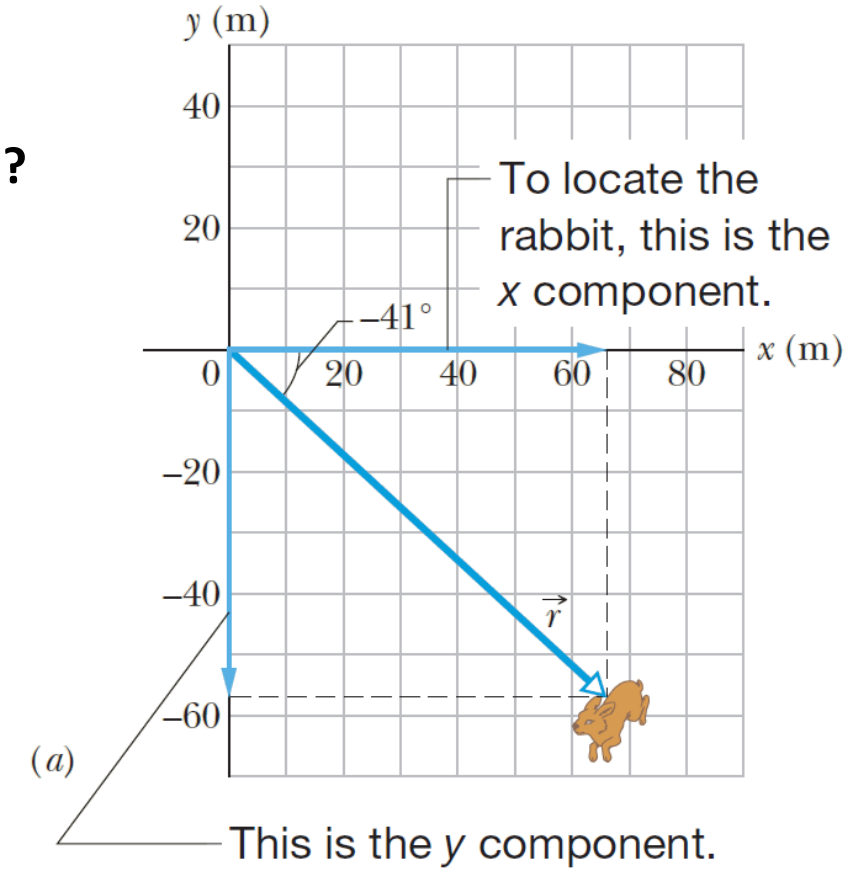
$$\Delta\vec{r} = \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath} + \Delta z\,\hat{k}$$

Example: A rabbit runs on the ground. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$x = -0.31t^2 + 7.2t + 28$ **and** $y = 0.22t^2 - 9.1t + 30$

At t = 15 s, what is the rabbit's position vector $\vec{r}$ in

a) unit vector notation and **b) in magnitude-angle notation?**



Solution:

a) To find the coordinates of the rabbit at t = 15 s, we substitute this value of t in the given equations:

$x = -0.31 \times (15)^2 + 7.2 \times (15) + 28 = 66$ m

$y = 0.22 \times (15)^2 - 9.1 \times (15) + 30 = -57$ m

$\therefore \vec{r} = 66\hat{\imath} - 57\hat{\jmath}$

This means that the rabbit will be at the point (66, – 57) at time t = 15 s

b) To find the vector $\vec{r}$ in magnitude-angle notation, we use the following relationship (between the vector and its components):

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(66)^2 + (-57)^2}$$

$$= \sqrt{7605}$$

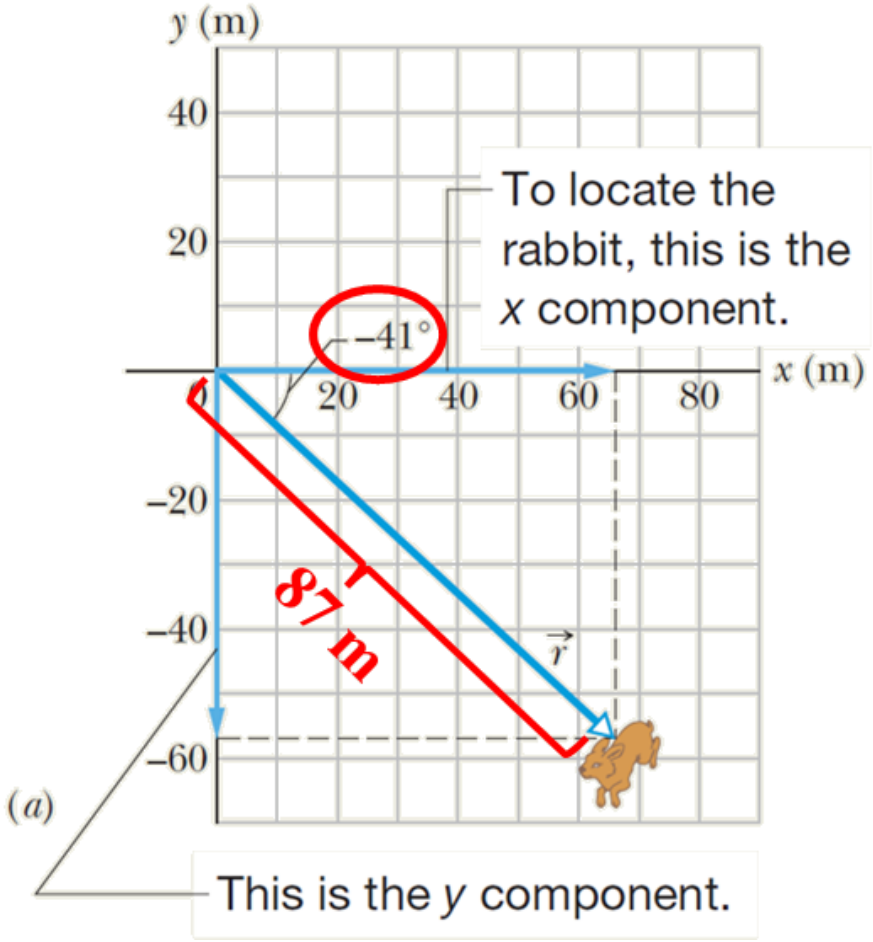
$$= 87 \text{ m}$$

The angle is calculated from:

$$\theta = tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = tan^{-1}\left(\frac{-57}{66}\right)$$

$$= tan^{-1}(-0.86) = -41°$$



The minus sign in the angle means that this angle is measured counterclockwise.

Average Velocity and Instantaneous Velocity

As we did in Chapter 2 (motion in one dimension), we define the average velocity $\vec{v}_{avg}$ and instantaneous velocity $\vec{v}$ but by using the displacement in 3 dimensions $\vec{r}$:

$$average\ volcity = \frac{displacement}{time\ interval}$$

$$\boxed{\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}}$$

But we know that $\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

$$\boxed{\therefore \vec{v}_{avg} = \frac{\Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}}$$

For example: If a particle moves through a displacement of (12 m) $\hat{i}$ + (3 m)$\hat{j}$ – (10 m) $\hat{k}$ in 2 s, then its average velocity during that move is

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(12\text{ m})\,\hat{i} + (3\text{ m})\,\hat{j} - (10\text{ m})\,\hat{k}}{2\ s} = (6\text{ m/s})\,\hat{i} + (1.5\text{ m/s})\,\hat{j} - (5\text{ m/s})\hat{k}$$

Average Velocity and Instantaneous Velocity

The **instantaneous velocity** $\vec{v}$ in 3 dimensions is:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d}{dt}\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$$

$$\vec{v} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

Average Acceleration And Instantaneous Acceleration

When a particle's velocity moves in 3 dimensions and changes from $\overrightarrow{v_1}$ to $\overrightarrow{v_2}$ in a time interval Δt, its **average acceleration** $\vec{a}_{avg}$ during Δt is:

$$average\ acceleration = \frac{change\ in\ velocity}{time\ interval}$$

$$\vec{a}_{avg} = \frac{\overrightarrow{v_2} - \overrightarrow{v_1}}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

The **instantaneous acceleration** in 3 dimensions (or **acceleration**) $\vec{a}$ is,

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d}{dt}\left(v_x \hat{\imath} + v_y \hat{\jmath} + v_z \hat{k}\right)$$

$$\boxed{\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k}}$$

$$\vec{a} = a_x \hat{\imath} + a_y \hat{\jmath} + a_z \hat{k}$$

Example: The position $\vec{r}$ of a particle moving in an *xy* plane is given by $\vec{r} = (2t^3 - 5t)\,\hat{i} + (6 - 7t^4)\hat{j}$, calculate (a) $\vec{r}$, (b) $\vec{v}$, and (c) $\vec{a}$ for $t = 2$ s. (d) What is the angle between the positive direction of the *x* axis and the particle's velocity at $t = 2$ s?

Solution:

a) At $t = 2$ s:

$\vec{r} = (2 \times 2^3 - 5 \times 2)\,\hat{i} + (6 - 7 \times 2^4)\hat{j} = 6\,\hat{i} - 106\,\hat{j}$

b) $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = \frac{d(2t^3 - 5t)}{dt}\hat{i} + \frac{d(6 - 7t^4)}{dt}\hat{j} = (6t^2 - 5)\,\hat{i} + (-28t^3)\,\hat{j}$

At $t = 2$ s:

$\vec{v} = (6 \times 4 - 5)\,\hat{i} - (28 \times 8)\,\hat{j} = 19\,\hat{i} - 224\,\hat{j}$

c) $\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = \vec{a} = \frac{d(6t^2 - 5)}{dt}\hat{i} + \frac{d(-28t^3)}{dt}\hat{j} = (12t)\,\hat{i} + (-84t^2)\,\hat{j}$

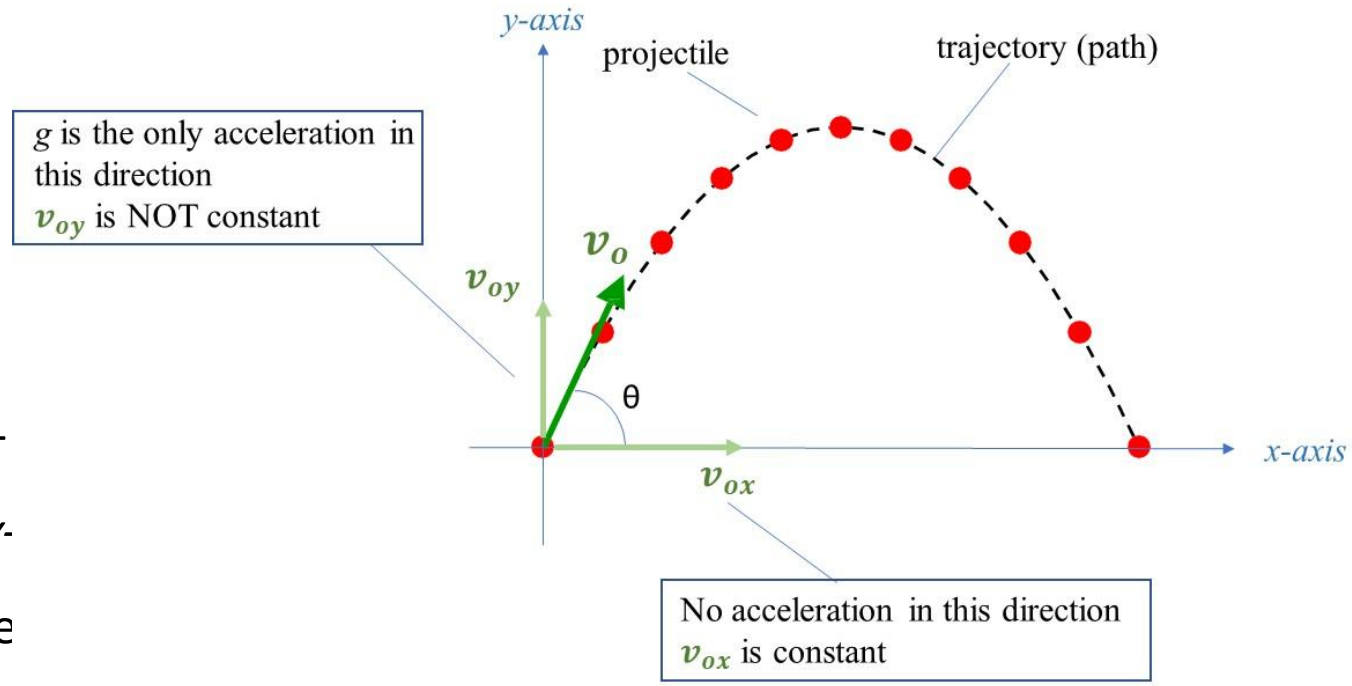
At $t = 2$ s:

$\vec{a} = (12 \times 2)\,\hat{i} - (84 \times 4)\,\hat{j} = 24\,\hat{i} - 336\,\hat{j}$

d) $\theta = tan^{-1}\left(\frac{v_y}{v_x}\right) = tan^{-1}\left(\frac{-224}{19}\right) = tan^{-1}(-11.8) = -85°$

Projectile Motion

- We consider a special case of two-dimensional motion: **projectile motion**
- In projectile motion, the particle moves with initial velocity $\overrightarrow{v_o}$
- The only acceleration in this motion is the free-fall acceleration g, which is downward (in the y-axis direction) $\Longrightarrow$ the velocity is changing in the y-axis direction (v_{oy}).
- No acceleration in the x-axis direction $\Longrightarrow$ no change in the speed in the x-axis direction (v_{ox})
- The only force acting on the projectile is the Earth's gravity



In this chapter, we neglect the air effect on the projectile motion

Projectile Motion

∵ the projectile motion has a constant acceleration (that is ***g*** in the *y*-axis direction)
∴ we can use the equations of motion that we learned in Chapter 2 to analyze this motion

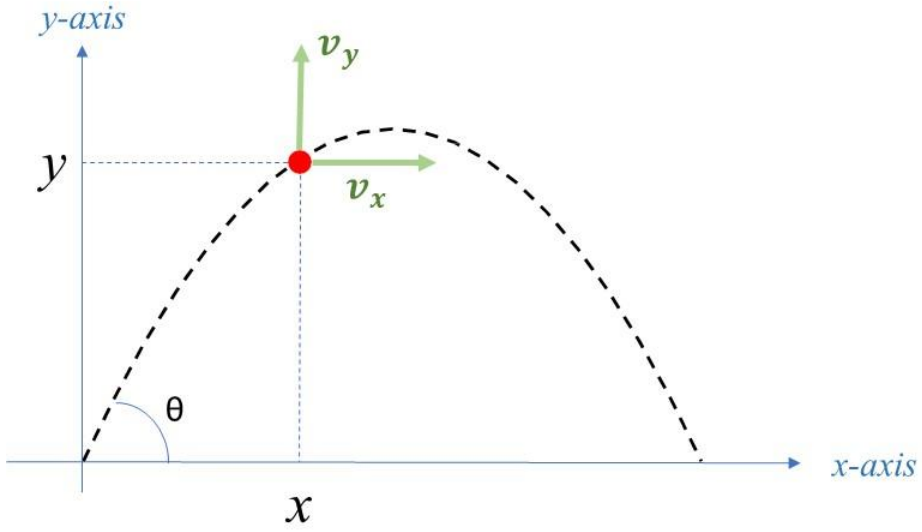
The Horizontal Motion (in the *x*-axis direction)

We can calculate the horizontal distance of the projectile at time *t* from the equation:

$$x = (v_o \cos\theta)\, t$$

and always the velocity in this direction is constant, and can be calculated form,

$$v_x = v_o \cos\theta$$



The Vertical Motion (in the *y*-axis direction)

We can calculate the vertical distance of the projectile at time *t* from the equation:

$$y = (v_o \sin\theta)\, t - \frac{1}{2} g t^2$$

and the velocity in this direction is NOT constant,

$$v_y = v_o \sin\theta - gt$$ and $$v_y^2 = (v_o \sin\theta)^2 - 2g(y - y_o)$$

t is the flight time (the time since the projectile started moving)
θ is the launch angle
v_o is the initial velocity

Projectile Motion

The Horizontal Range

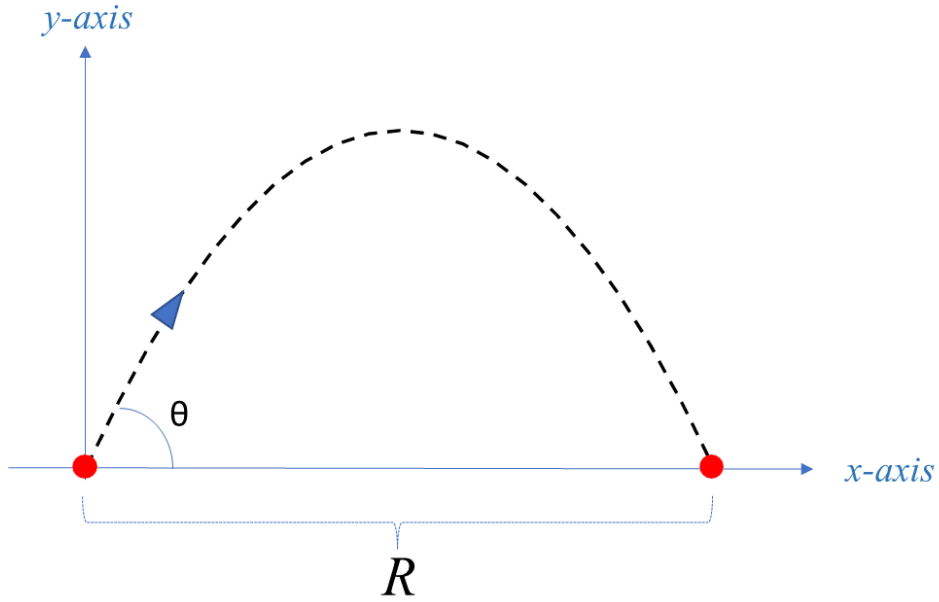
- The *horizontal range R* of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched).
- *R* can be calculated from,

$$R = \frac{v_o^2}{g} \sin(2\theta)$$

where,

θ is the launch angle

v_o is the initial velocity



- The horizontal range *R* is maximum for a launch angle of 45°

Example: An object is launched from ground at a velocity of 40 m/s in a direction making an angle of 50° upward with the horizontal.

a) What is the maximum height reached by the object?

b) What is the object's total flight time (between launch and touching the ground agian)?

c) What is the object's horizontal range (maximum *x* above ground)?

d) What is the object's position (horizontal and vertical coordinates) at $t = 2$ s

e) What is the object's horizontal and vertical velocity at $t = 2$ s

Solution:

a) We have given that $v_o = 40$ m/s and $\theta = 50°$

Remember that at maximum height, the projectile (the object) stops briefly

$\Longrightarrow v_y = 0$ m/s

First, we calculate the time for reaching the maximum height:

$$v_y = v_o \, sin\theta - gt$$

$$0 = 40 \, sin50° - 9.8 \, t$$

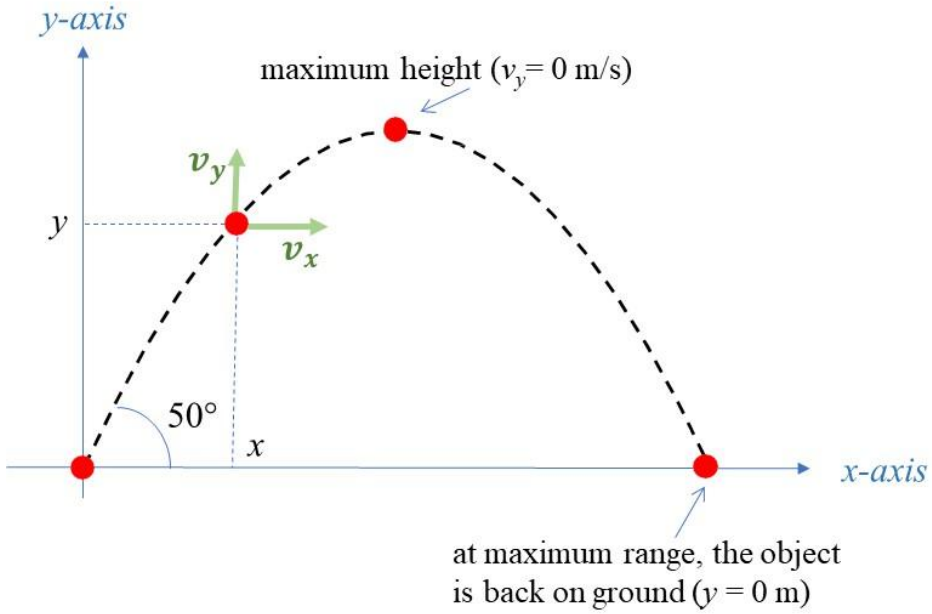
$$9.8 \, t = 40 \, sin50°$$

$$t = \frac{40 \, sin50°}{9.8} = 3.1 \text{ s}$$

Now, we can calculate the maximum height (using t for maximum height):

$$y = (v_o \, sin\theta) \, t - \frac{1}{2} g t^2$$

$$= (40 \times sin50°) \times 3.1 - \frac{1}{2} \times 9.8 \times 3.1^2 = 47.9 \text{ m}$$



b) When the object finishes the flight, it will reach the ground again $\Longrightarrow$ $y = 0$ m

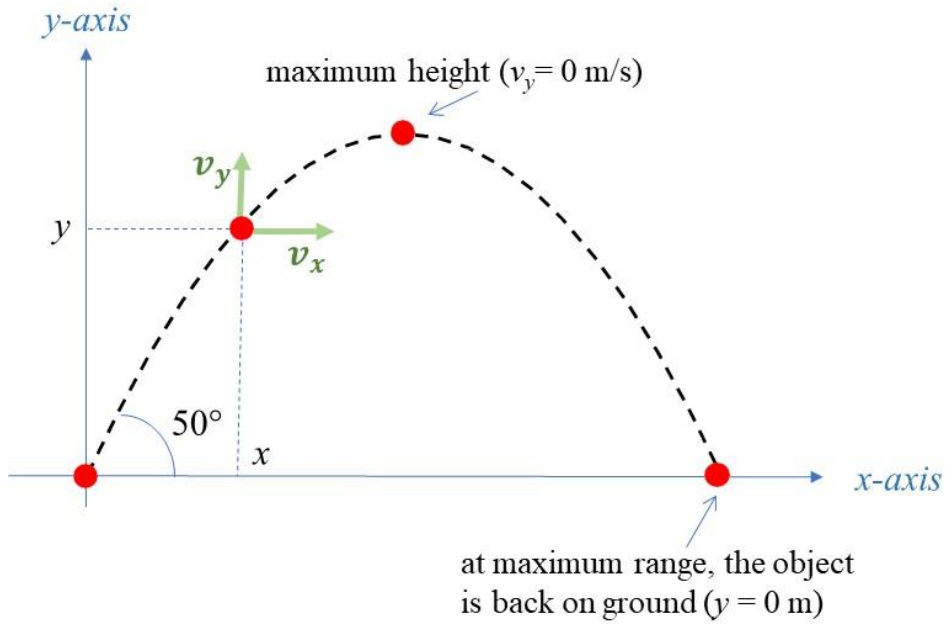
$$y = (v_o \, sin\theta)\, t - \frac{1}{2} g t^2$$

$$0 = (40 \times sin50°)t - \frac{1}{2} \times 9.8 \times t^2$$

$$\frac{1}{2} \times 9.8 \times t^2 = (40 \times sin50°)t$$

$$\frac{1}{2} \times 9.8 \times t = 40 \times sin50$$

$$t = \frac{2 \times 40 \times sin50}{9.8} = 6.25 \text{ s}$$



c) $R = \frac{v_o^2}{g} \sin(2\theta)$

$$R = \frac{40^2}{9.8} \sin(2 \times 50) = \frac{1600}{9.8} \sin(100) = 160.8 \text{ m}$$

d) We know: $x = (v_o \cos\theta)\, t$ and $y = (v_o \sin\theta)\, t - \frac{1}{2} g t^2$

So at t = 2 s:

$$x = (v_o \cos\theta)\, t = 40 \times \cos 50° \times 2 = 51.4 \text{ m}$$

$$y = (v_o \sin\theta)\, t - \frac{1}{2} g t^2 = 40 \times \sin 50° \times 2 - \frac{1}{2} \times 9.8 \times 2^2 = 41.7 \text{ m}$$

e) We know that $v_x = v_o \cos\theta$ and $v_y = v_o \sin\theta - gt$

So at t = 2 s:

$$v_x = v_o \cos\theta = 40 \cos 50° = 25.7 \frac{\text{m}}{\text{s}}$$

$$v_y = v_o \sin\theta - gt = 40 \sin 50° - 9.8 \times 2 = 11 \text{ m/s}$$

Note that the velocity in the x direction (v_x) does NOT change with time (it is constant during the whole flight).

UNIFORM CIRCULAR MOTION

- A particle is in **circular motion** if it travels in a circular path

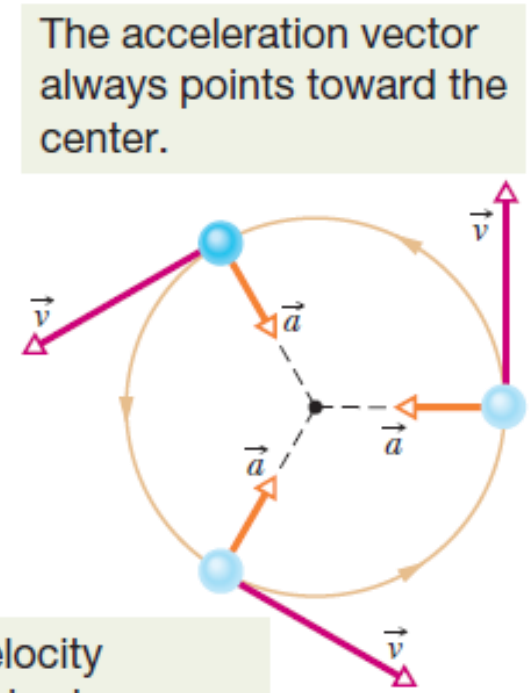
  A particle is in **uniform motion** if it travels at constant (*uniform*) speed.

  ∴ A particle is in **uniform circular motion** if it travels in a circular path at constant (*uniform*) speed.

- Although the speed does not vary, *the particle is **accelerating*** because the velocity changes in direction.
- This acceleration is always directed *radially inward.* Because of this, the acceleration is called a **centripetal acceleration.**
- The magnitude of this acceleration $\vec{a}$ is

$$a = \frac{v^2}{r}$$

where,
a is the centripetal acceleration
v is the "linear" velocity of the particle
r is the radius of the circular path

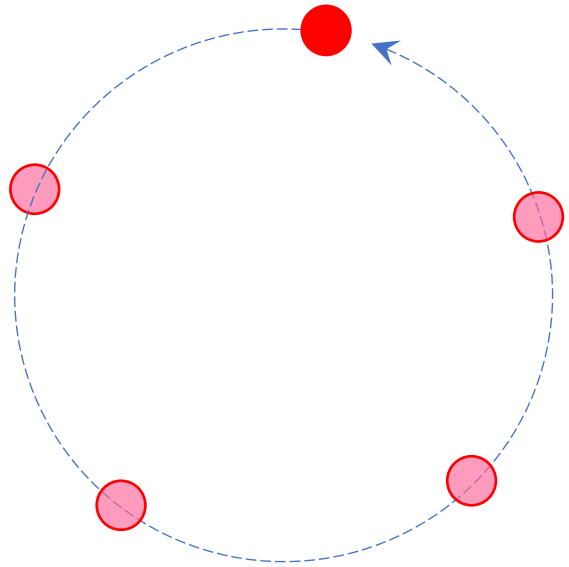


UNIFORM CIRCULAR MOTION

T is called the *period of revolution,* or simply the *period,* of the motion. It is the time for a particle to go around the path exactly once

$$T = \frac{2\pi r}{v}$$

where,
T is the period
v is the "linear" velocity of the particle
r is the radius of the circular path



Example: a car moves in circular road with a velocity of 5 m/s. the radius of the road path is 30 m. Find a) the centripetal acceleration of the car, b) the period of this motion.

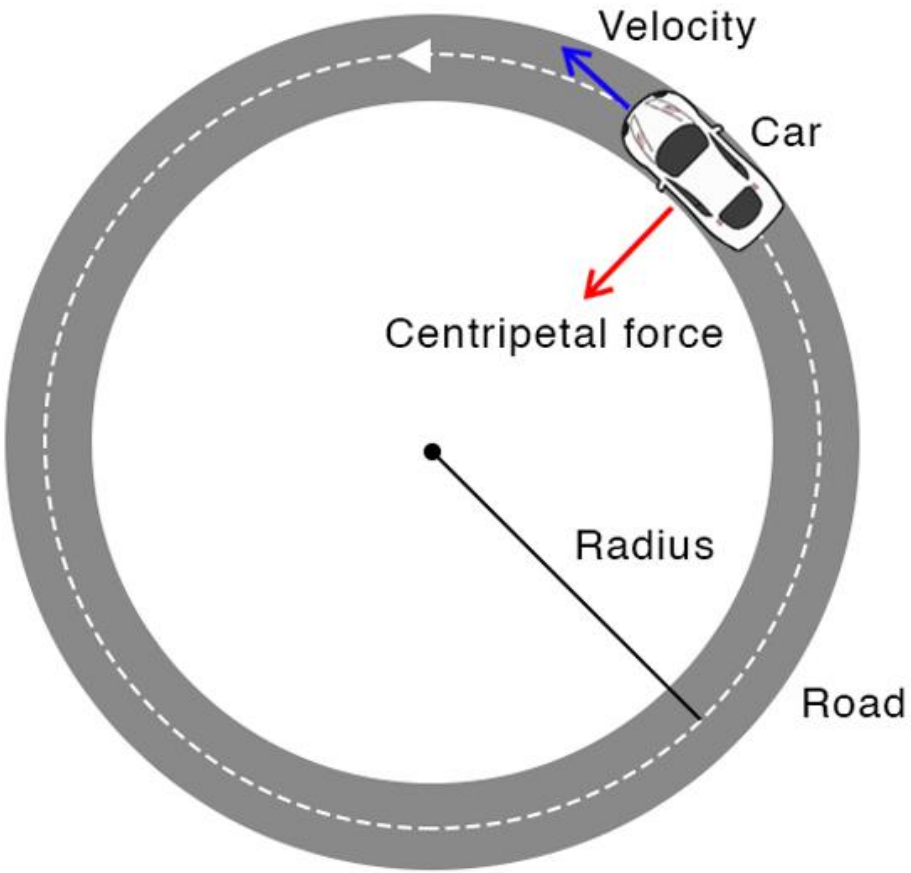
Solution:

a) $a = \frac{v^2}{r}$

$= \frac{5^2}{30} = 0.83\ \mathrm{m/s^2}$

b) $T = \frac{2\pi r}{v}$

$= \frac{2\times 3.14\times 30}{5} = 37.7\ \mathrm{s}$



Chapter 5

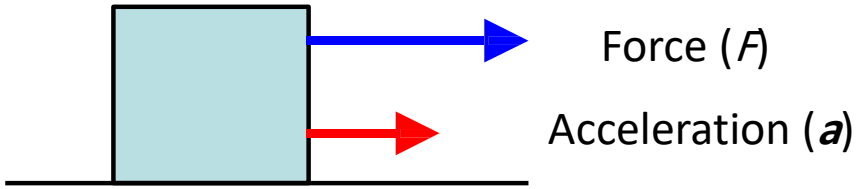
Force and Motion I

Newtonian Mechanics

- The relation between a force applied to a body, and the acceleration that occurred to that body is called *Newtonian Mechanics.*
- We shall study the three primary laws of motion.
- For very high speeds of bodies, Newtonian mechanics is replaced by Einstein's special theory of relativity.
- For very small bodies (e.g. electrons), Newtonian mechanics is replaced by quantum mechanics.
- We assume in this chapter that all surfaces are slippery or **frictionless surface,** i.e. no friction between the moving object and the surface

Force

- Before we study Newton's laws, we must know what *Force* means:
  - A force causes an object to accelerate.
  - A force acts on an object to change its velocity.



- The SI unit of force is "Newton" or N

  1 N = the force needed to accelerate 1 kg of mass by 1 m/s^2

- The force is a vector quantity (it has magnitude and direction)

Force

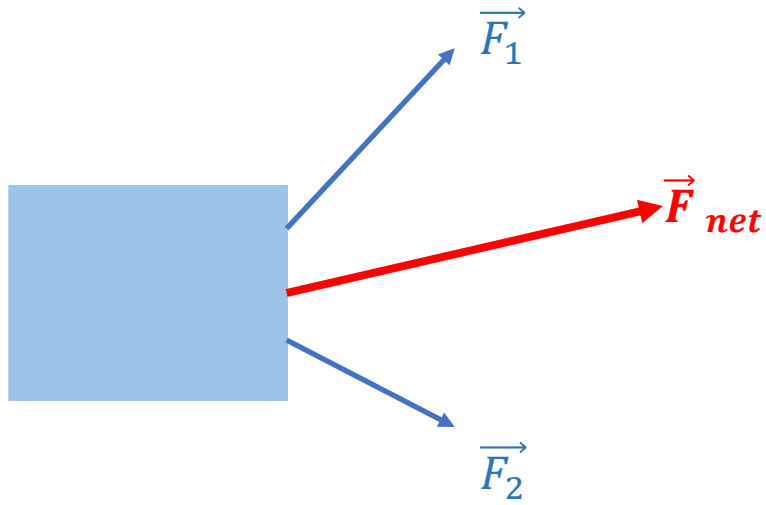
∴Assume that 2 forces ($\overrightarrow{F_1}$ and $\overrightarrow{F_2}$) are acting on the object

- The object will move because of all these 2 forces

  $\Longrightarrow$ we must "**add**" the forces together

when we add forces together, we must use the ***vector sum*** rules

(from Chapter 3)



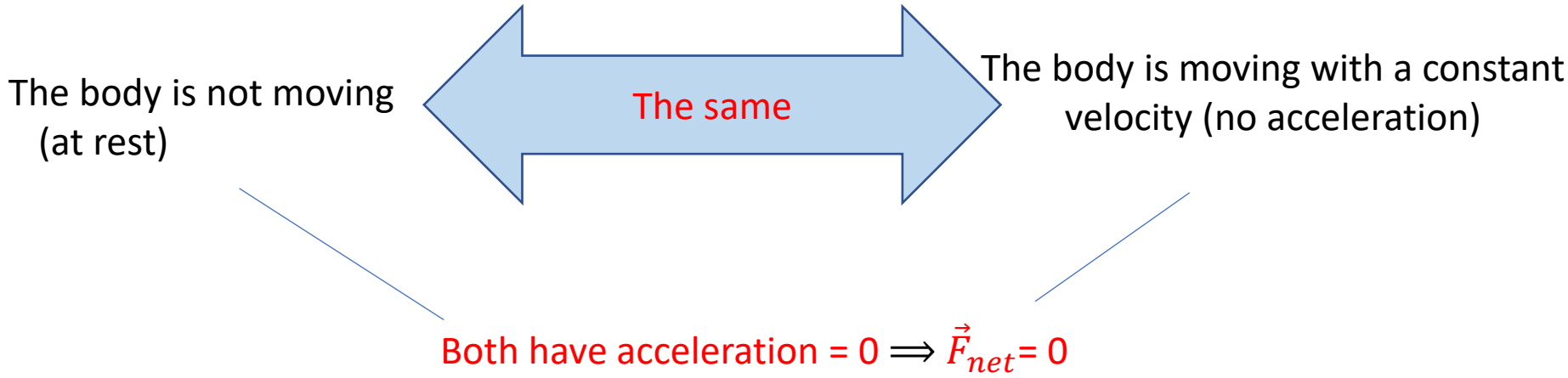
- The ***net force*** is called ***resultant force*** (F_{net})
- The net force has the same effect as that of all the individual forces together.
- This force addition is called the **principle of superposition for forces**

Newton's First Law

- If no **force** acts on a body, the body's velocity cannot change; that is, the body cannot accelerate.

OR

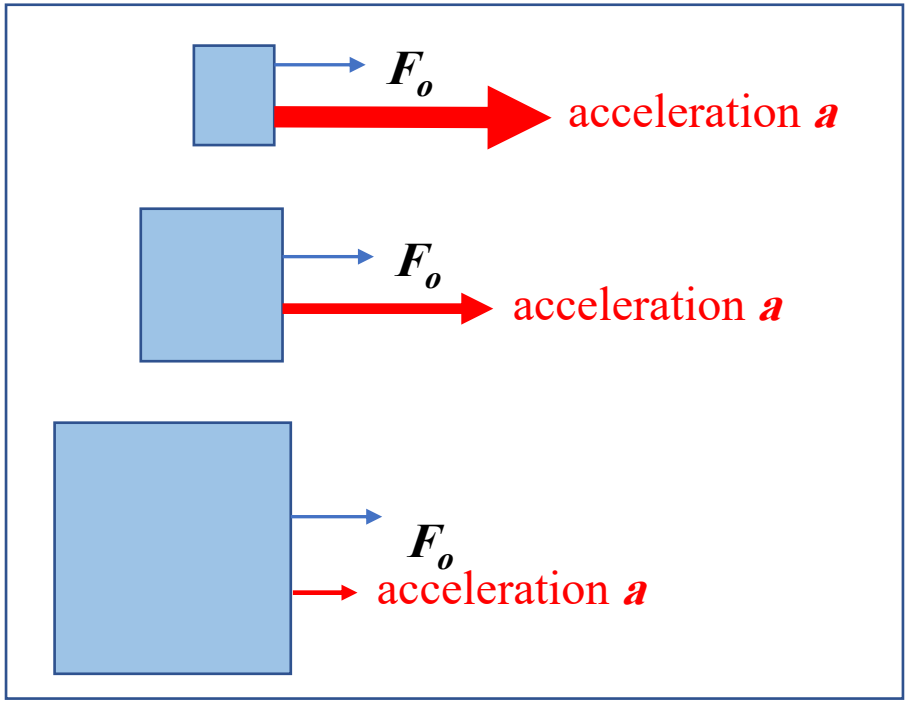
- If no ***net* force** acts on a body ($\vec{F}_{net}$= 0), the body's velocity cannot change; that is, the body cannot accelerate.



Mass

- The object with the larger mass is accelerated less
- The acceleration is inversely related to the mass
- If the same force F_o acts on several masses, the acceleration a will decrease when the mass m increases (as shown in the figure)

We can say that *the mass of a body is the characteristic that relates a force on the body to the resulting acceleration.*



Newton's Second Law

The net force on a body is equal to the product of the body's mass and its acceleration.

$$\vec{F}_{net} = m\vec{a}$$

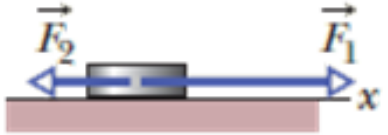
where,

F_{net} is the net force acting on the body
m is the mass
a is the acceleration

In 3 dimensions, both $\vec{F}$ and $\vec{a}$ have 3 components (in *x*, *y* and *z* directions):

$F_{net,x} = ma_x$, $F_{net,y} = ma_y$, $F_{net,z} = ma_z$

The SI unit of force is "Newton" or N $1\ \text{N} = (1\ \text{kg})(1\ \text{m/s}^2) = 1\ \text{kg m/s}^2$

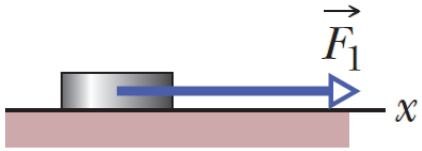


The Free-body diagram

system is included in the sketch. In case of two (or more) forces act on a body, we draw a free-diagram, in which the body is represented by a point (●), and each force acting on that body represented by a vector arrow.



Example: a) A body is moving on a frictionless surface as shown. The mass of the body is 0.2 kg, and the force F_1 acting on it is 4 N. What is the acceleration of the body?



Solution:

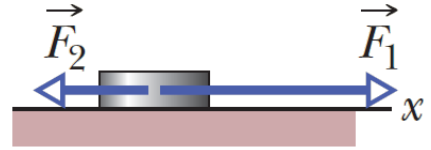
We know from Newton's second law that:



$$F = m\,a$$

$$\therefore a = \frac{F}{m} = \frac{4}{0.2} = 20\ \mathrm{m/s^2}$$

b) Now, another force F_2 = 2 N acts on the body as shown. What is the acceleration of the body?



Solution:

We must find the resultant (net) force F_{net} of both F_1 and F_2 before we can use Newton's second law.

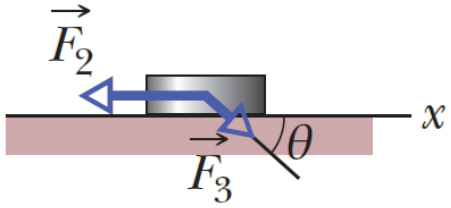


$$F_{net} = F_1 - F_2 = 4 - 2 = 2\ \mathrm{N}$$

Newton's second law will be:

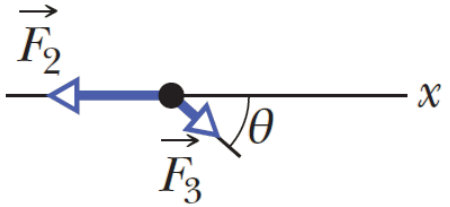
$$F_{net} = m\,a \quad \Rightarrow \quad a = \frac{F_{net}}{m} = \frac{2}{0.2} = 10\ \mathrm{m/s^2}$$

c) Now, a new force F_3 = 1 N and directed at angle θ = 30° acts on the body as shown. What is the acceleration of the body?



Solution:

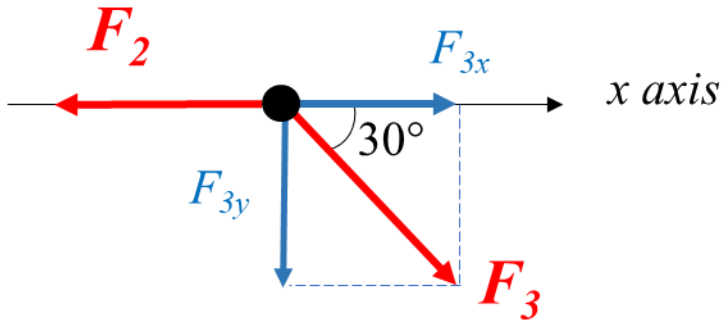
We must first analyze the force F_3 (and consider only the *x*-axis component)



then we find the resultant force:

$$F_{3x} = F_3\, cos30 = 1 \times cos30 = 0.87 \text{ N}$$

$$\therefore F_{net} = F_{3x} - F_2 = 0.87 - 2 = -1.13 \text{ N}$$



$$F_{net} = m\,a \quad \Rightarrow \quad a = \frac{F_{net}}{m} = \frac{-1.13}{0.2} = -5.67 \text{ m/s}^2$$

The minus sign means the net force accelerates the body in the negative direction of the *x* axis

Example: Two horizontal forces act on a 2 kg block that moves over a frictionless surface which lies in an *xy* plane. One force is $\vec{F}_1 = 3\hat{\imath} + 4\hat{\jmath}$. Find the acceleration of the block in unit-vector notation when the other force is a) $\vec{F}_2 = -3\hat{\imath} - 4\hat{\jmath}$ and b) $\vec{F}_2 = -3\hat{\imath} + 4\hat{\jmath}$

Solution:

a) First, we must find $\vec{F}_{net}$ by adding $\vec{F}_1$ and $\vec{F}_2$ (using the component method):

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = (3\hat{\imath} + 4\hat{\jmath}) + (-3\hat{\imath} - 4\hat{\jmath})$$
$$= (3 - 3)\hat{\imath} + (4 - 4)\hat{\jmath} = 0$$

Now, we use Newton's second law:

$$F_{net} = m\,a \quad \Rightarrow \quad a = \frac{F_{net}}{m} = \frac{0}{2} = 0 \text{ m/s}^2$$

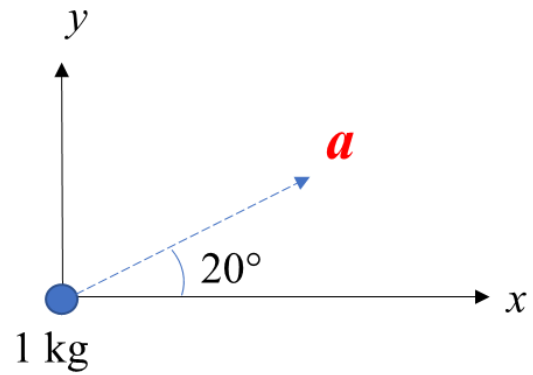
b)

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = (3\hat{\imath} + 4\hat{\jmath}) + (-3\hat{\imath} + 4\hat{\jmath})$$
$$= (3 - 3)\hat{\imath} + (4 + 4)\hat{\jmath} = 8\,\hat{\jmath}$$

Now, we use Newton's second law:

$$F_{net} = m\,a \quad \Rightarrow \quad a = \frac{F_{net}}{m} = \frac{8\,\hat{\jmath}}{2} = 4\,\hat{\jmath} \text{ m/s}^2$$

Example: If a 10 kg body has an acceleration of 2 m/s² at 20° to the positive direction of an *x-axis*, what are (a) the *x* component and (b) the *y* component of the net force acting on the body, and (c) what is the net force in unit-vector notation?

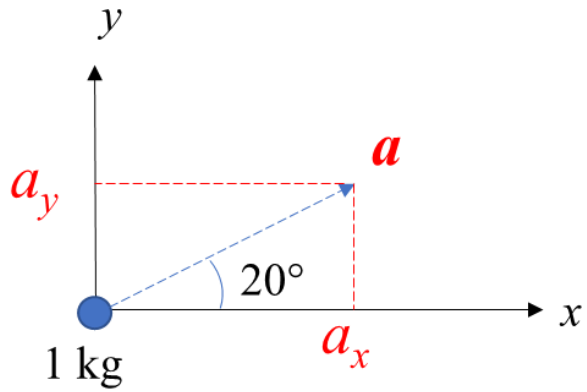


Solution:

We need to analyze the acceleration a to its *x* and *y* components:

$a_x = a \cos\theta = 2 \times \cos 20° = 1.88$ m/s²

$a_y = a \sin\theta = 2 \times \sin 20° = 0.68$ m/s²



Now we can calculate the components of the net force:

a) $F_x = m\,a_x = 10 \times 1.88 = 18.8$ N

b) $F_y = m\,a_y = 10 \times 0.68 = 6.8$ N

c) The net force: $\vec{F} = 18.8\hat{\imath} + 6.8\hat{\jmath}$

Some Particular Forces

The Gravitational Force

- The Earth is always pulling masses *toward the Earth center*
- This force is called **gravitational force** $\vec{F}_g$:

with magnitude $F_g = mg$

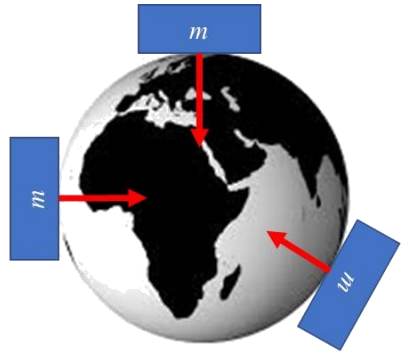
and directed **<u>downward</u>** regardless the surface orientation

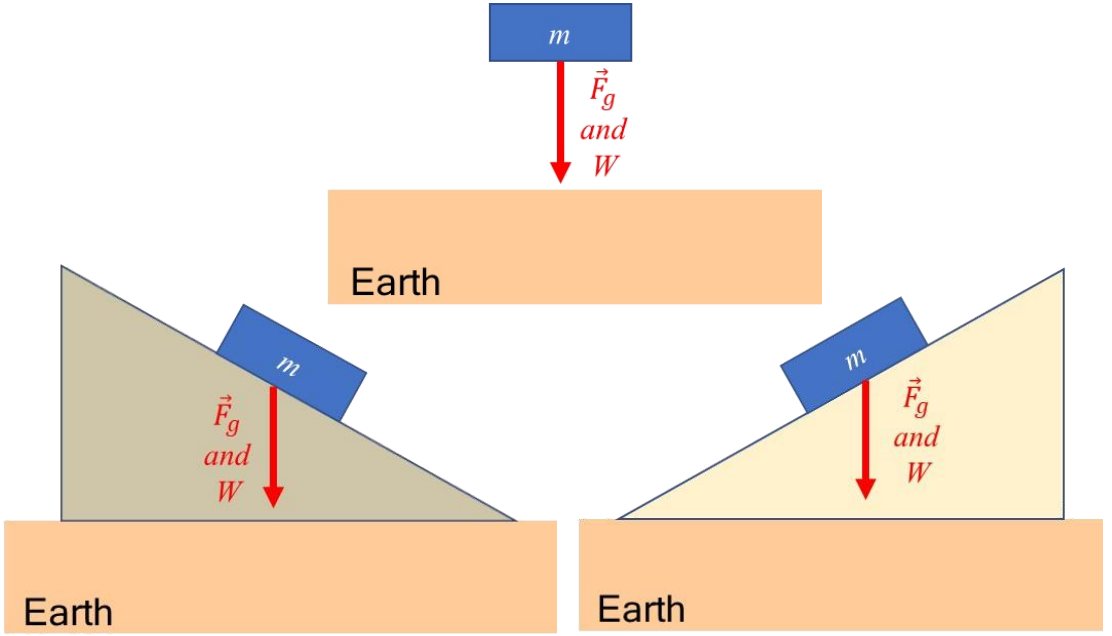
where *m* is the mass of the object

g is the gravitational acceleration = 9.8 m/s^2

- The weight *W* of a body equas the magnitude of the gravitational force F_g on the body :

$$\vec{F}_g = W \quad \text{or} \quad W = mg$$





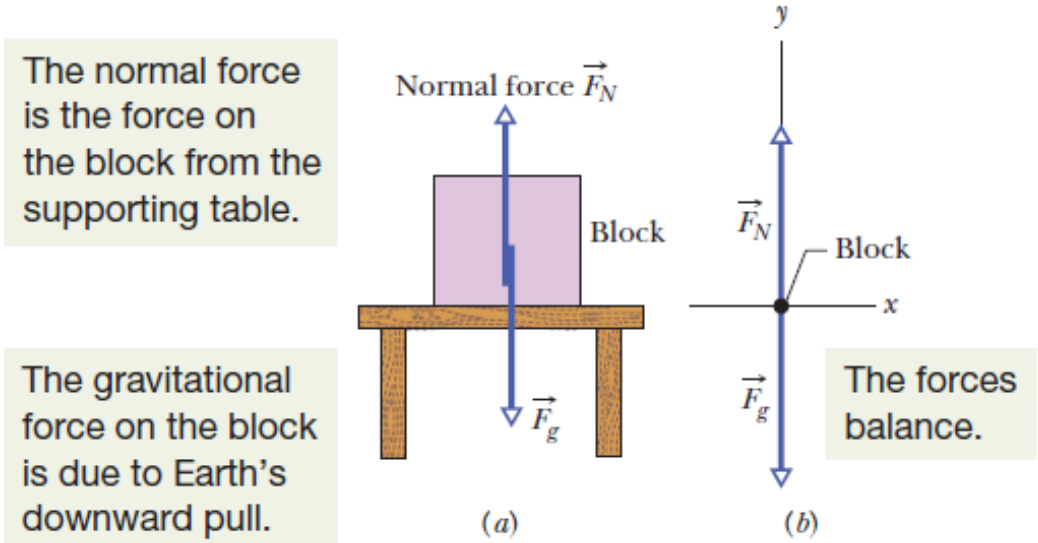
$\vec{F}_g$ and *W* are *<u>always</u>* directed vertically downward

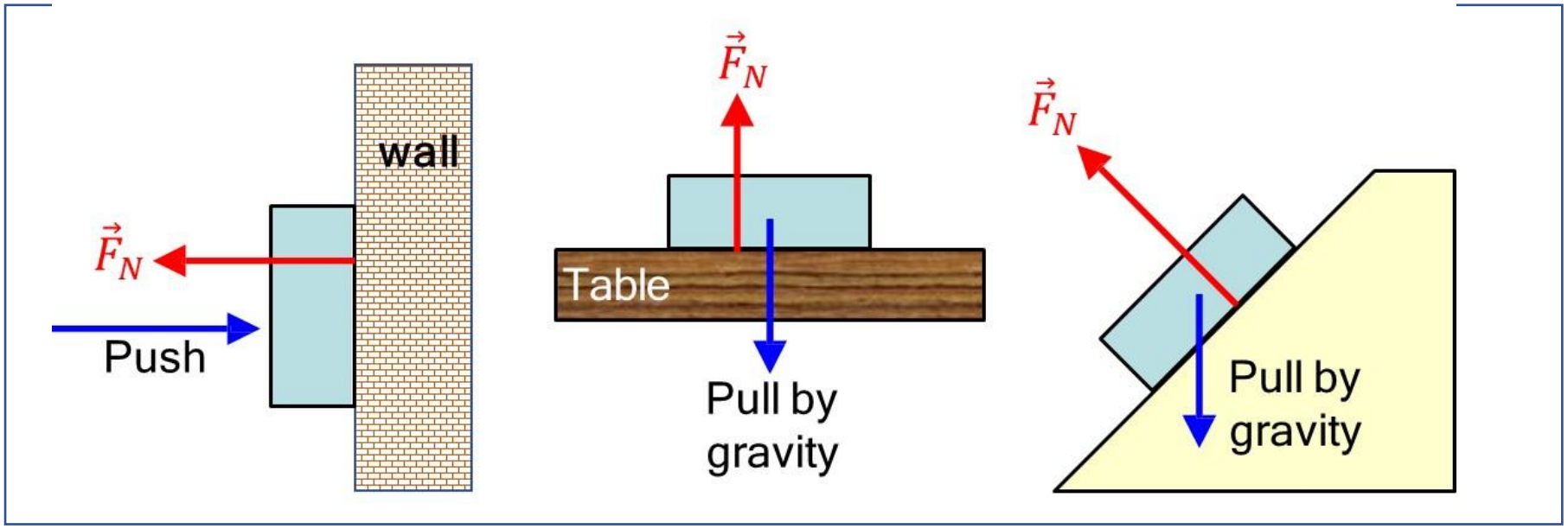
The Normal Force

When an object is placed on a surface, the surface pushes the object with a normal force $\vec{F}_N$

The block in the figure balances because the gravitational force (acting downward) equals the normal force (acting upward).

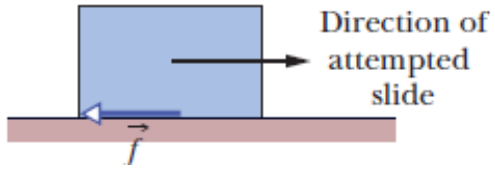
$\vec{F}_N$ is always perpendicular to the surface.





Friction Force

When an object slides over a surface, the motion is resisted by the force of friction $\vec{f}$.

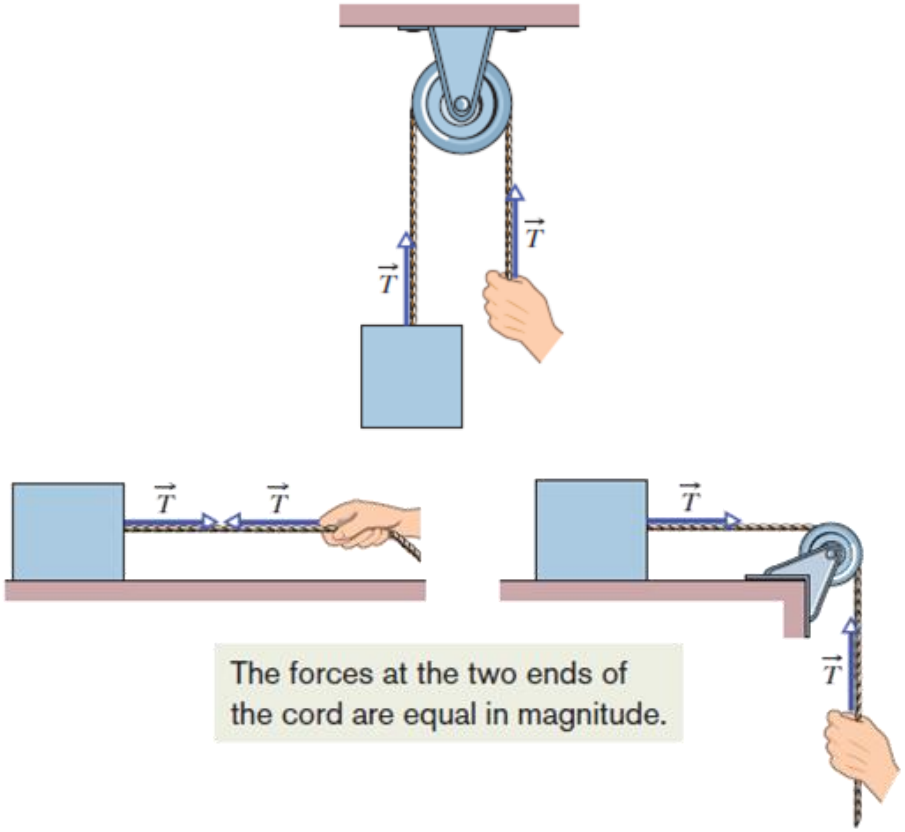


This force is directed along the surface, opposite the direction of the motion.

This friction force will be discussed in detail in the next chapter.

Tension Force

- When a is attached to a body and pulled, the cord applies a force $\vec{T}$ on the body.
- This force is directed away from the body and along the cord.
- The tension in the cord is the magnitude T of the force on the body.



Newton's Third Law

- To every action there is an equal and opposite reaction
- When two bodies interact, the force "$\vec{F}_{AB}$" which body A exerts on body B (the action force) is equal in magnitude and opposite in direction to the force "$\vec{F}_{BA}$" which body B exerts on body A (the reaction force).

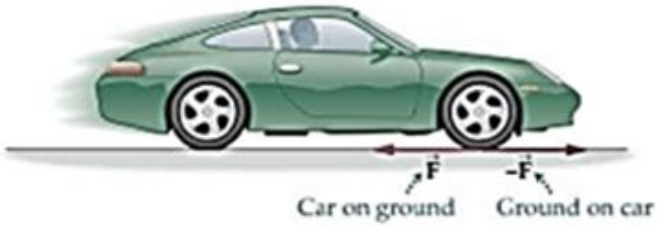
$$F_{AB} = F_{BA} \qquad \text{(equal magnitudes)}$$

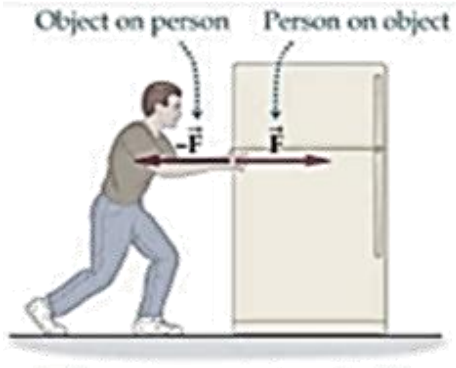


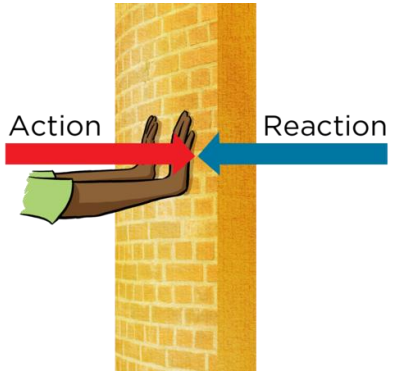
or as the vector relation,

$$\vec{F}_{AB} = -\vec{F}_{BA} \qquad \text{(equal magnitudes and opposite directions)}$$

- The action force and the reaction force act on different objects.



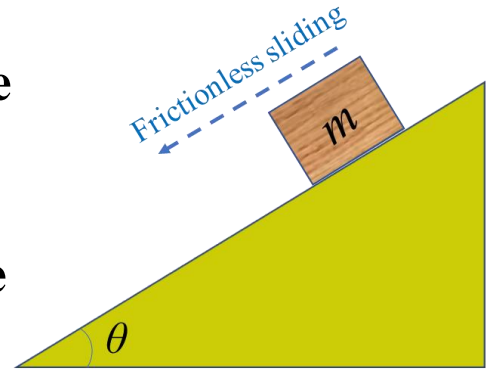




Example: A block of mass 8.5 kg placed at a frictionless inclined surface with the angle $\theta = 30°$. Find

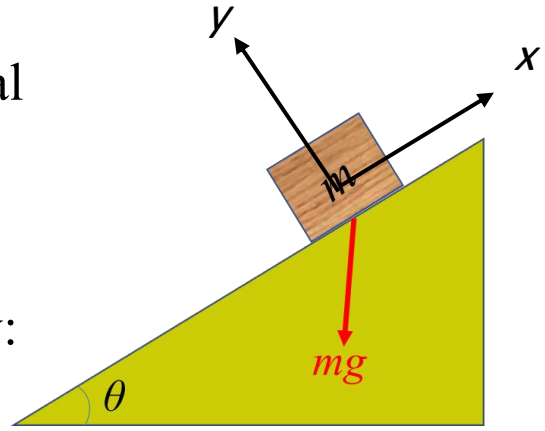
(a) the normal force acting on the block.

(b) If the block starts moving from rest on this surface, find the magnitude of the resulting acceleration of the block.



Solution:

∵ there is no friction ⟹ the only force acting on the box is its weight (or gravitational force $F_g = mg$)

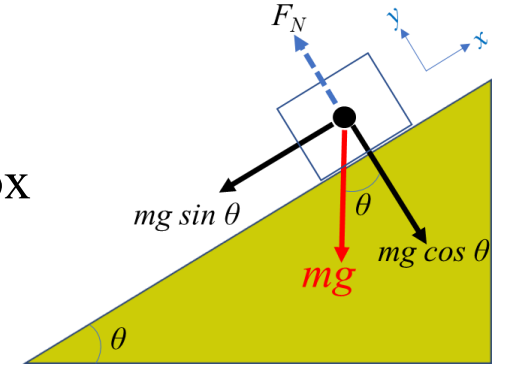


We first choose suitable coordinates for the system Choose the x-axis parallel to the inclined surface. Next, draw the free-body diagram to analyze the weight of the body:

$F_{gx} = F_g \sin\theta = m g \sin\theta$ (this is the force that causes the motion)

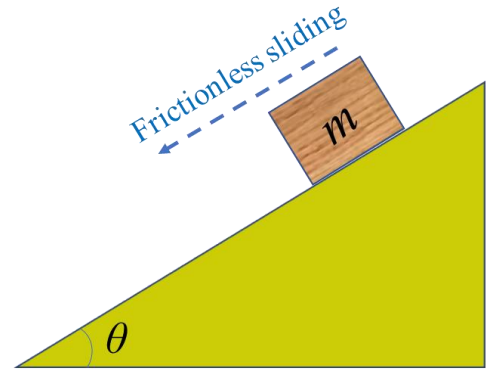
$F_{gy} = F_g \cos\theta = m g \cos\theta$ (this is the force that is against the surface)

F_N is perpendicular to the surface and equal in magnitude to the "push" that the box applies to the surface:



a) The normal force F_N

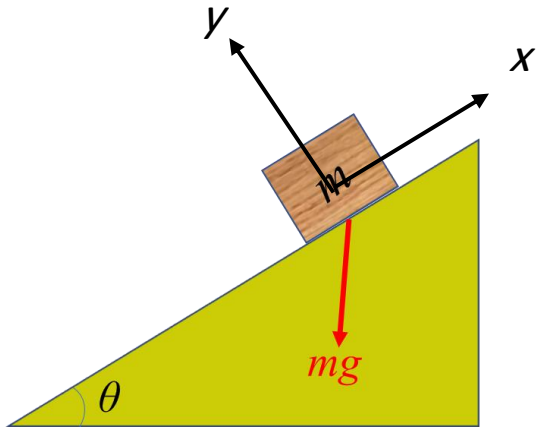
$$F_N = F_{gy} = m\,g\,cos\,\theta = 8.5 \times 9.8 \times \cos 30° = 72.1 \text{ N}$$

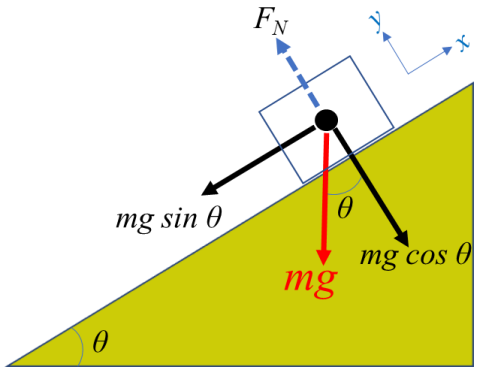


b) The force causing the motion is $F_{gx} = F_g\,sin\,\theta = m\,g\,sin\,\theta$

$\Longrightarrow$ the magnitude of acceleration in this direction:

$$a = \frac{mg\ \sin\theta}{m} = g\,sin\,\theta = 9.8 \times \sin 30° = 4.9 \text{ m/s}^2$$





Example: A 500 kg rocket is accelerated at a constant rate from rest to 444 m/s in 1.8 s. What is the magnitude of the required net force?

Solution:

To calculate the force, we need to calculate the acceleration first.

∵ the acceleration is constant ⟹ we can use the equation :

$$v = v_o + at$$

$$444 = 0 + a\,(1.8)$$

$$a = \frac{444}{1.8} = 246.6\ \text{m/s}^2$$

Now we can calculate the force:

$$F = ma$$

$$= 500 \times 246.6 = 1.2 \times 10^5\ \text{N}$$

Example: A car that weighs 1.3×10^4 N is initially moving at 11.1 m/s when the brakes are applied and the car is brought to stop in 15 m. Assuming the force that stops the car is constant, find (a) the magnitude of that force and (b) the time required for the change in speed.

Solution:
We must first find the mass and the acceleration of the car:
From the weight W of the car we can calculate the mass: $W = mg \Longrightarrow m = W/g = 1.3 \times 10^4 / 9.8 = 1327$ kg
We can find the acceleration from:

$$v^2 = v_o^2 + 2ax$$

$$0 = 11.1^2 + 2 \times a \times 15$$

$$-11.1^2 = 30\, a \qquad \Rightarrow \qquad a = -\frac{11.1^2}{30} = -4.1 \text{ m/s}^2$$

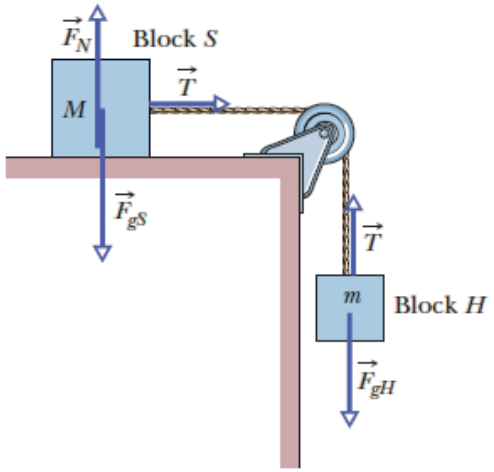
The minus sign means that the car was decelerating

a) $F = m\, a = 1327 \times 4.1 = 5.4 \times 10^3$ N

b) $v = v_o + at \qquad \Rightarrow \qquad 0 = 11.1 - 4.1\, t$

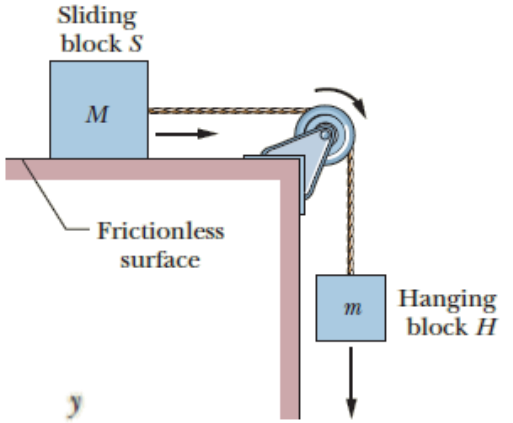
$$-11.1 = -4.1\, t \qquad \Rightarrow \qquad t = \frac{-11.1}{-4.1} = 2.7 \text{ s}$$

Example: a block S with mass M =3.3 kg. The block is free to move along a horizontal frictionless surface and connected by a cord to a second block H, with mass m =2.1 kg. The cord has negligible mass. The hanging block H falls as the sliding block S accelerates to the right. Find (a) the acceleration of block S, (b) the acceleration of block H, and (c) the tension in the cord.



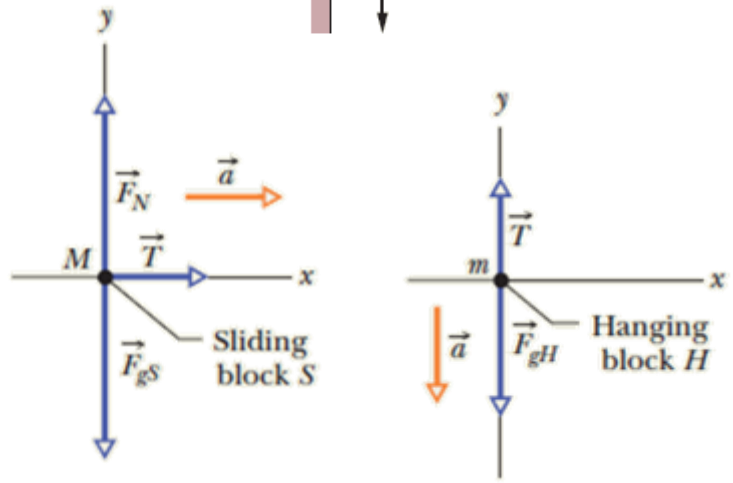
Solution:

- All the forces on the system have to be specified first as shown.



- Then, a free-body diagram is drawn on each mass, and equations of motion are written.

- We note here that the horizontal acceleration of mass M equals the vertical acceleration of mass m because both are attached to each other by a cord.

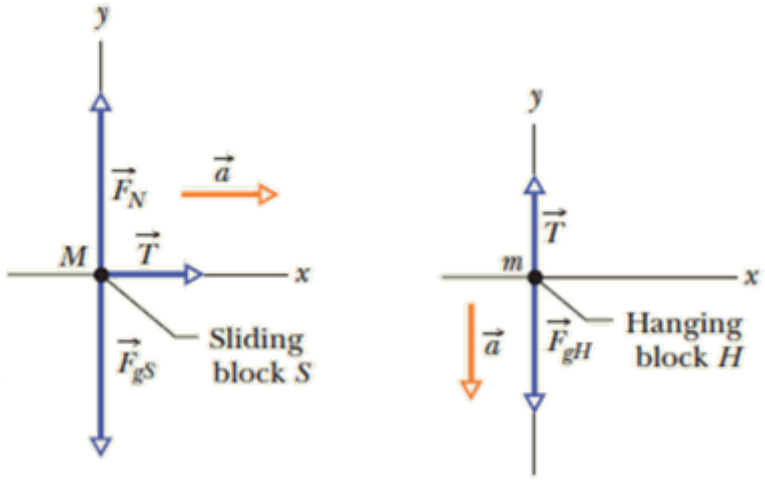


- Applying Newton's second law on the sliding block S :

$$F_{net,x} = Ma \Longrightarrow \qquad T = Ma$$

- Applying Newton's second law on the hanging block H:

$F_{net,y} = T - F_{gH} = -ma \quad (-\downarrow)$ but F_{gH} (weight of the mass m) $= mg$



$$\therefore T - mg = -ma \quad \Longrightarrow \quad Ma - mg = -ma$$

$$mg = ma + Ma \quad \Longrightarrow \quad mg = a(M + m)$$

$$\therefore a = \frac{m}{M + m} g$$

$a = \frac{2.1\ kg}{3.3\ kg + 2.1\ kg}\ 9.8\ m/s^2 = 3.8\ m/s^2$ which is the acceleration of both M and m

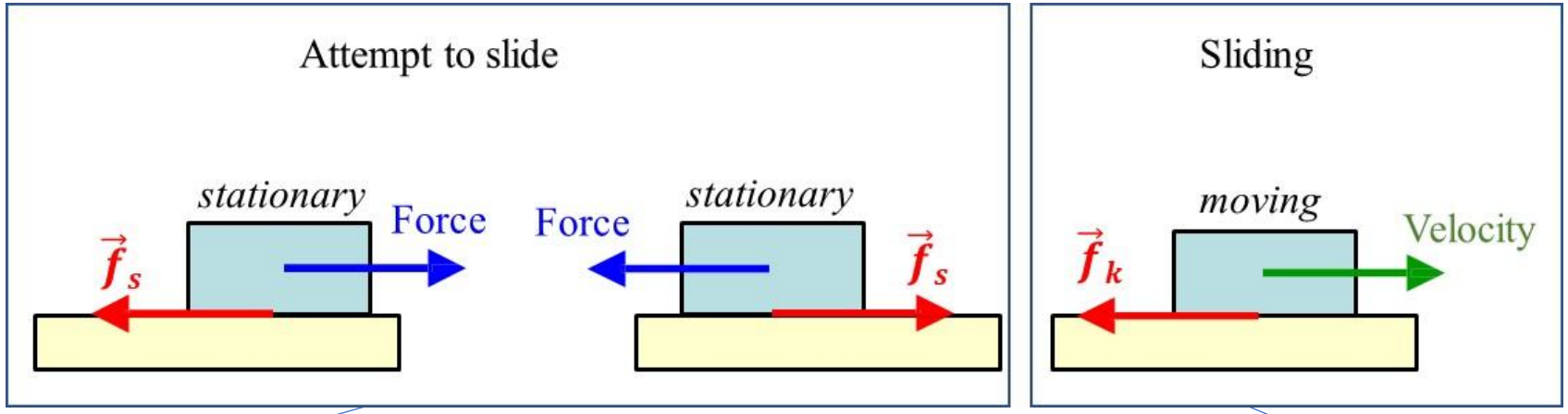
$$T = Ma = 3.3kg \times 3.8 \frac{m}{s^2} = 12.5\ N$$

Friction

- Friction is a force between **two surfaces** that are **sliding**, **or trying to slide**, across each other.
- It prevents our shoes from slipping on the pavement when we walk.
- It stops car tyres from slipping on the road.
- Sometimes we want to reduce friction. For example, we use oil to reduce the friction between the moving parts inside a car engine.
- Many other examples in our daily life: driving of a vehicle on a surface, applying brakes to stop a moving vehicle, walking on the road, writing on a notebook/ blackboard, drilling a nail into a wall, lighting a matchstick, etc…

Friction

- If we either slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface.
- The resistance is because of the **frictional force** $\vec{f}$ or simply **friction.**
- Friction force is always:
  1) directed along the surface,
  2) opposite the direction of the intended motion



If the body does not slide, the frictional force is a **static frictional force** $\vec{f}_s$

If there is sliding (motion), the frictional force is a **kinetic frictional force** $\vec{f}_k$

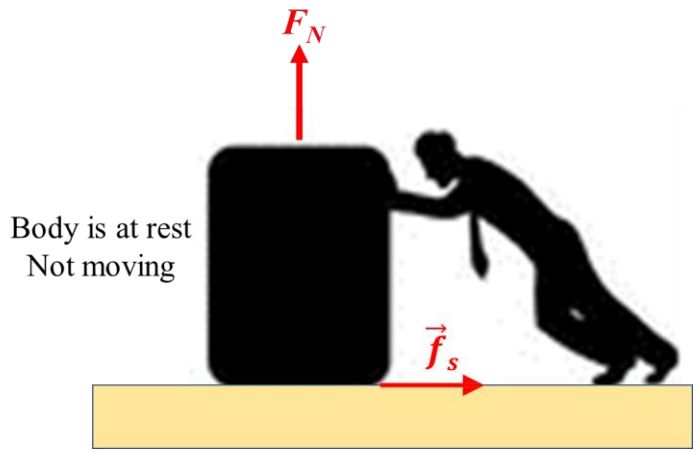
Properties of Friction

Static friction force $\vec{f}_s$:

The magnitude of $\vec{f}_s$ has a maximum value $\vec{f}_{s,max}$ that is given by

$$\vec{f}_{s,max} = \mu_s F_N$$

where,
μ_s is the coefficient of static friction
F_N is the normal force

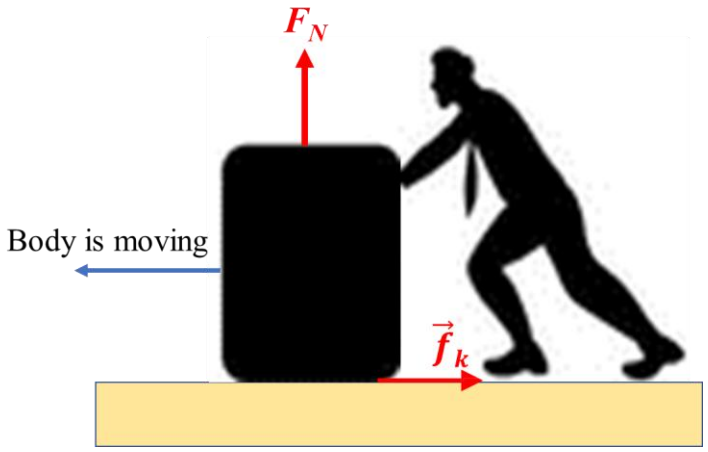


Kinetic friction force $\vec{f}_k$:

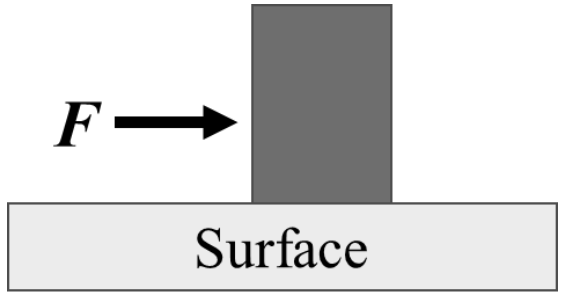
The magnitude of $\vec{f}_k$ is given by

$$\vec{f}_k = \mu_k F_N$$

where,
μ_k is the coefficient of kinetic friction
F_N is the normal force



Example: A 2.5 kg block is initially at rest on a horizontal surface. A horizontal force F = 60 N is applied to the block as shown in the figure. The coefficients of friction for the block and surface are μ_s = 0.40 and μ_k = 0.25. Determine the magnitude of the friction force acting on the block.



Solution:

The block is pushing the surface with its weight $W = F_g = mg = 2.5 \times 9.8 = 24.5$ N

$\therefore$ the normal force on the block $F_N = 24.5$ N

There are 2 types of friction forces:

1) The static fraction force:

$\vec{f}_{s,\,max} = \mu_s F_N = 0.4 \times 24.5 = 9.8$ N

Note: $\because$ the applied force $F > f_s \Longrightarrow$ the block will move (if F was less than f_s, the block will stay at rest because the force F is not enough to overcome the friction)

2) The kinetic friction force appears after the object moves:

$\vec{f}_k = \mu_k F_N = 0.25 \times 24.5 = 6.1$ N

Exercise 5:

1. Given the forces $\vec{F}_1 = (106\ N)\hat{x} + (85\ N)\ \hat{y}$ and $\vec{F}_2 = (-70\ N)\hat{x} + (55\ N)\ \hat{y}$ applied on an object of mass 12 kg. Find:

a. The resultant force.

b. The components of the acceleration vector.

2. A box of mass m=35 kg is pulled with a force of magnitude F=140 N horizontal floor (Figure 1). Assume friction is negligible. Calculate:

a. The acceleration of the box.

b. The magnitude of the normal force

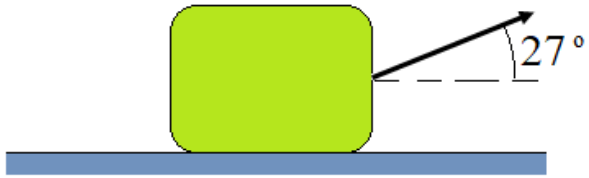


Figure 1

Chapter 7

Kinetic Energy and Work

Kinetic Energy

- **Kinetic energy** K is the energy that the object gain when it moves.
- The faster the object moves, the greater is its kinetic energy.
- When the object is not moving (stationary), its kinetic energy is zero.

- For an object of mass m and speed v, then its kinetic energy is:

$$K = \frac{1}{2}mv^2$$

- The SI unit of kinetic energy is $kg.m^2/s^2$ or Joule (J)
- Kinetic energy is a scalar quantity (has no direction)

For example, if a mass of 3 kg is moving with a speed of 2 m/s, then the kinetic energy of this mass is:

$$K = 0.5 \times 3 \times 2^2 = 6 \text{ J}$$

Example: a car of mass 1500 kg starts moving from rest with a constant acceleration of 2 m/s^2. What is the car's kinetic energy after a period of time of 30 s?

Solution:

The velocity of the car after $t = 30$ s is:

$$v = v_o + at$$

$$v = 0 + 2 \times 30 = 60 \text{ m/s}$$

The kinetic energy is:

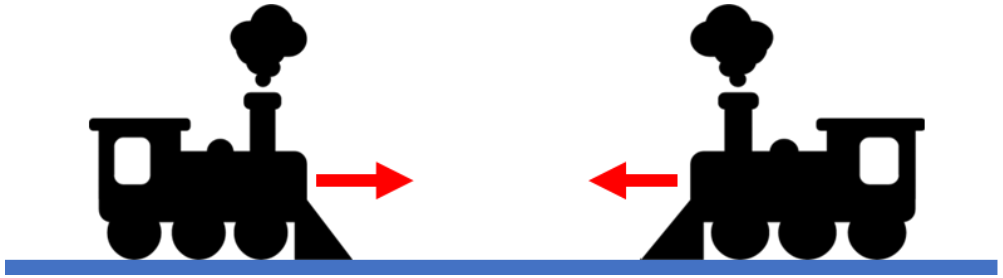
$$K = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 1500 \times 60^2$$

$$= 2.7 \times 10^6 \text{ J}$$

Example: Two identical trains move in opposite direction (on the same line) with a speed of 40.8 m/s. Each train weighs 1.2×10^6 N. What is the total kinetic energy of the two trains just before the collision?

Solution:



First we calculate the mass of each train:

$$W = mg$$

$$m = \frac{w}{g} = \frac{1.2\times10^6}{9.8} = 1.22 \times 10^5 \text{ kg}$$

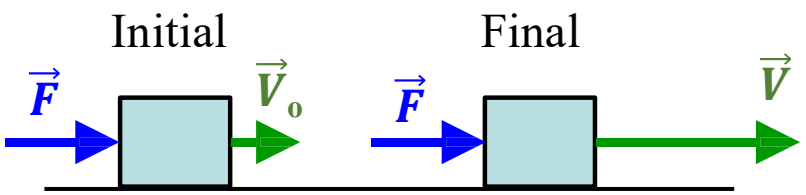
Now we find the total kinetic energy of the ***two*** trains:

$$K_{total} = \left(\frac{1}{2}mv^2\right) + \left(\frac{1}{2}mv^2\right)$$
$$= mv^2$$
$$= 1.22 \times 10^5 \times 40.8^2$$
$$= 2 \times 10^8 \text{ J}$$

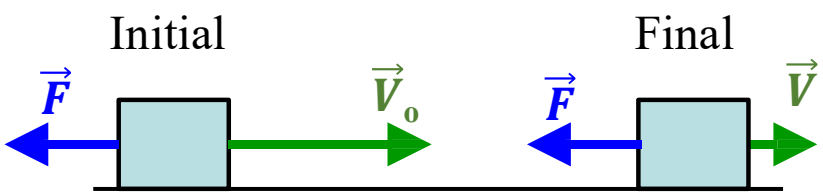
Note: because kinetic energy is a scalar quantity, the direction of the motion (speed) of each train does not affect the value of its energy.

Work

If you apply a force to an object, you can either *increase* or *decrease* its velocity



- The force increases the kinetic energy of the object.
- The force has **transferred** energy **to** the object from you.
- This is **positive work**.



- The force decreases the kinetic energy of the object.
- The force has **transferred** energy **from** the object to you.
- This is **negative work**.

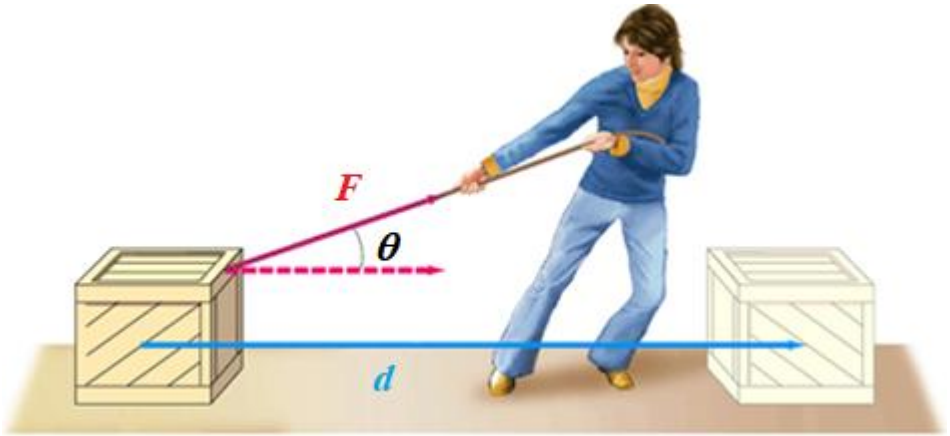
- Work is a scalar quantity
- The SI unit of work is Joule (J)

work ⟷ same ⟷ energy

If you apply a constant force $\vec{F}$ on an object, and thus, the object makes a displacement $\vec{d}$, then you can calculate the work W done by this force from the equation:

$$W = \vec{F} \cdot \vec{d}$$ (note: this is scalar product)

$$W = Fd\cos\theta$$



where,

W is the work done by the force F

F is the force

d is the displacement

θ is the angle between $\vec{F}$ and $\vec{d}$

Note that: to do work, there must be displacement.

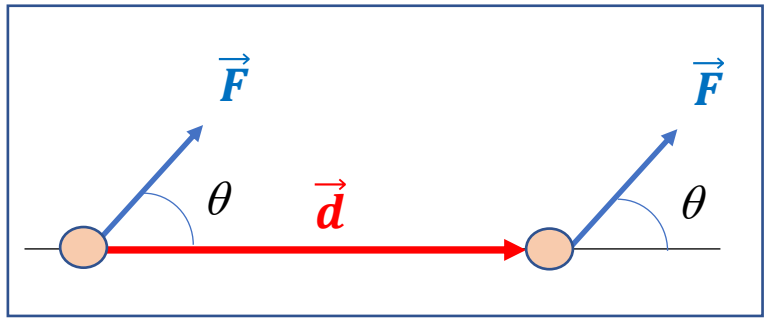
If you apply large force F but no displacement happens $\Longrightarrow$ no work



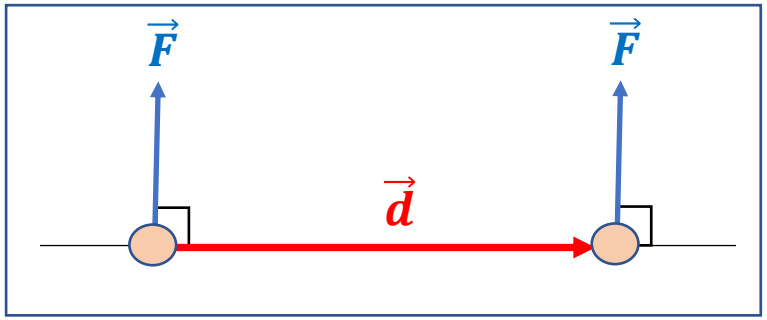
$\because W = Fd\,cos\theta$

$\therefore$ depending on the value of θ, the work W can be:

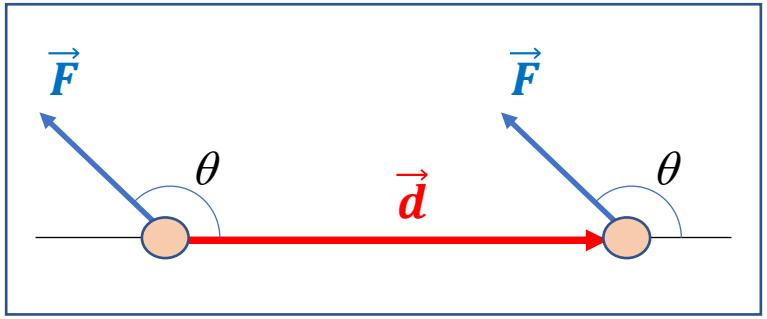
positive
($0° \leq \theta < 90°$)



zero
($\theta = 90°$)

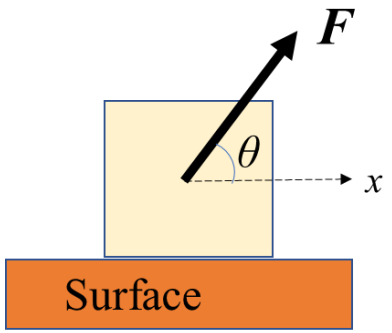


negative
positive ($180° \geq \theta > 90°$)



Example: a force F = 15 N acts on a box as shown. Find the work done by this force when the box moves for a horizontal distance of 3 m (in the positive direction of x-axis) if (a) θ = 50° and (b) θ = 115°

Solution:

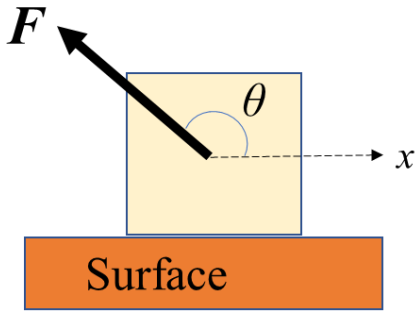


a) $W = Fd\ cos\theta$

$= 15 \times 3 \times \cos 50°$

$= 28.9$ J

The positive W means that the force is increasing the kinetic energy of the box $\Longrightarrow$ the speed of the box is increasing.



b) $W = Fd\ cos\theta$

$= 15 \times 3 \times \cos 115°$

$= -19$ J

The minus sign of W means that the force is lowering (decreasing) the kinetic energy of the box $\Longrightarrow$ the speed of the box decreases.

Example: A block is pushed through a displacement (in meter) $\vec{d} = 15\,\hat{\imath} - 12\,\hat{j}$ along a straight path by a force (in Newton) $\vec{F} = 210\,\hat{\imath} - 150\,\hat{\jmath}$. How much work does the force do on the block during the displacement?

Solution:

We can solve this problem by using the scalar product of the vectors (from Chapter 3):

$$W = \vec{F} \cdot \vec{d}$$

$$= \left(15\,\hat{\imath} - 12\,\hat{j}\right) \cdot (210\,\hat{\imath} - 150\,\hat{\jmath})$$

$$= [15 \times 210] + [(-12) \times (-150)]$$

$$= 3150 + 1800$$

$$= 4950\ \text{J}$$

Work–Kinetic Energy Theorem

- Assume there is an object with initial velocity v_i (initial kinetic energy K_i), then you apply work W on it. Your work will change the object's velocity to v_f (final kinetic energy K_f)
- The **work-kinetic energy theorem** states that:

The change in the kinetic energy = the net work applied

$$\begin{pmatrix}\text{kinetic energy after the}\\ \text{net work is done, } K_f\end{pmatrix} - \begin{pmatrix}\text{kinetic energy before the}\\ \text{net work, } K_i\end{pmatrix} = (\text{the net work done, } W)$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W$$

$$K_f - K_i = W$$

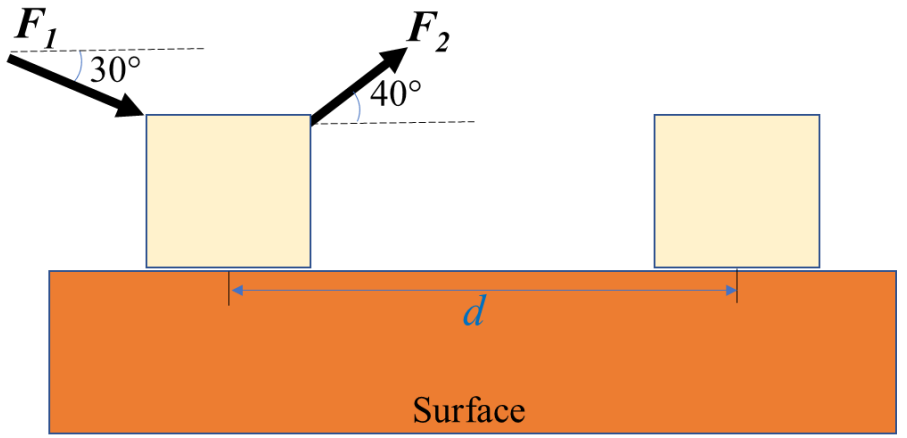
$$\Delta K = W$$

The change in kinetic energy ΔK (and also the work done W) can be *positive* or *negative*

If the net work done on a object is positive, then the object's kinetic energy increases by the amount of the work.

If the net work done is negative, then the object's kinetic energy decreases by the amount of the work.

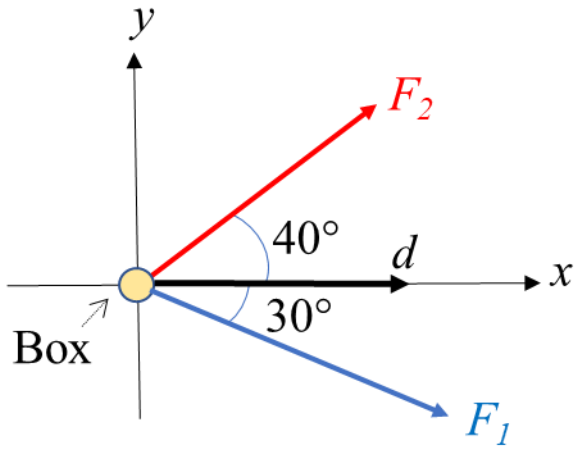
Example: two constant forces act on a box of 225 kg as shown: $F_1 = 12$ N at an angle of 30° and $F_2 = 10$ N at an angle of 40°. These forces moved the box a displacement $d = 8.5$ m on a frictionless surface. (a) What is the net work done on the box by these forces during the displacement? (b) The box is initially stationary. What is its speed v_f at the end of the 8.5 m displacement?



Solution:

a) The done by F_1 is:

$$W_1 = F_1\, d \cos 30° = 12 \times 8.5 \times \cos 30° = 88.3 \text{ J}$$



The done by F_2 is:

$$W_2 = F_2\, d \cos 40° = 10 \times 8.5 \times \cos 40° = 65.1 \text{ J}$$

$$\therefore\ W_{total} = W_1 + W_2 = 88.3 + 65.1 = 153.4 \text{ J}$$

b) The speed of the box changes because its kinetic energy is changed when energy is transferred to it by the forces F_1 and F_2

We can use the work-kinetic energy theorem to find the box's final velocity:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

But the box was stationary before the forces acted on it $\Longrightarrow v_i = 0$

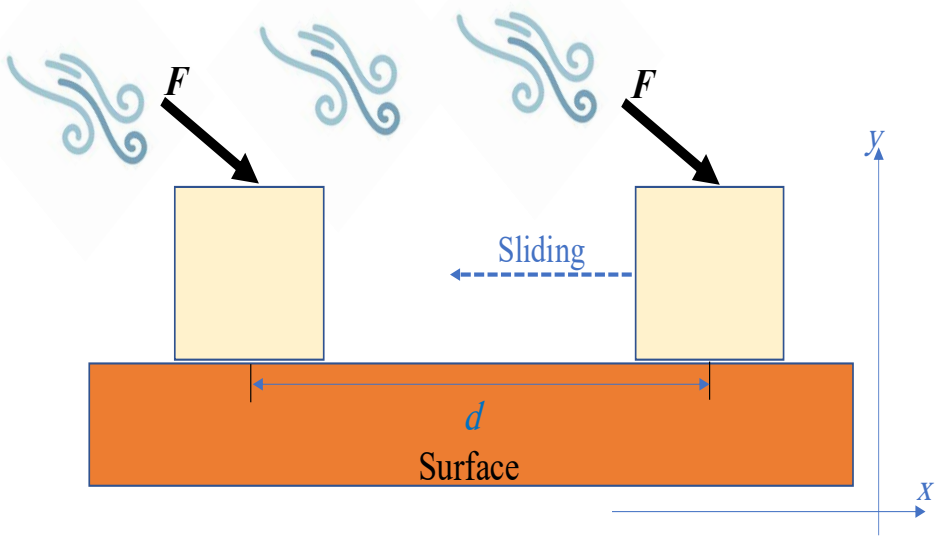
$$W = \frac{1}{2}mv_f^2 - 0$$

$$W = \frac{1}{2}mv_f^2$$

$$v_f^2 = \frac{2W}{m}$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 153.4}{225}} = 1.17 \text{ m/s}$$

Example: a box is sliding in the negative direction of *x*-axis on a frictionless surface. Suddenly, wind starts blowing on the box with a constant force as shown, where this force $\vec{F} = 2\hat{\imath} - 6\hat{\jmath}$. (a) Find the work done by this force during the displacement of 3 m (this means $\vec{d} = -3\hat{\imath}$). (b) the box has kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?



Solution:

a)

$$W = \vec{F} \cdot \vec{d}$$
$$W = [(2\hat{\imath} - 6\hat{\jmath})] \cdot [(-3\hat{\imath})]$$
$$= (2 \times -3) + (-6 \times 0)$$
$$= -6 \text{ J}$$

The minus sign means that the force does a negative work of 6 J on the box. This means that the force transferred 6 J of energy from the kinetic energy of the box.

b) The work-kinetic energy theorem:

$$K_f - K_i = W$$
$$K_f = W + K_i$$
$$K_f = (-6) + 10 = 4 \text{ J}$$

Less kinetic energy means that the box has been slowed

Power

- the **power** due to the force is the time **rate at which work is done** by that force.
- If a force does work W in an amount of time Δt, then the **average power** P_{avg} is

$$P_{avg} = \frac{W}{\Delta t}$$

- The **instantaneous power** P of doing work is

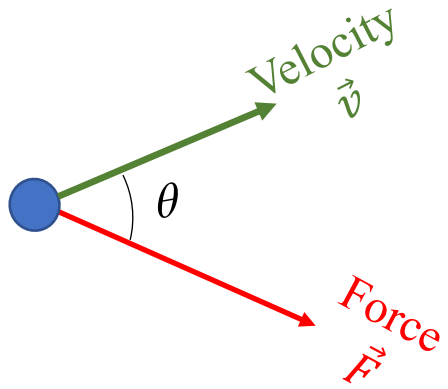
$$P = \frac{dW}{dt}$$

- Power is a scalar quantity
- The SI unit of the power is J/s or Watt (W)

If a force $\vec{F}$ acts on an object that moves with an instantaneous velocity $\vec{v}$, then we can calculate the **instantaneous power** P:

$$P = \vec{F} \cdot \vec{v}$$

$$P = Fv\cos\theta$$



where,

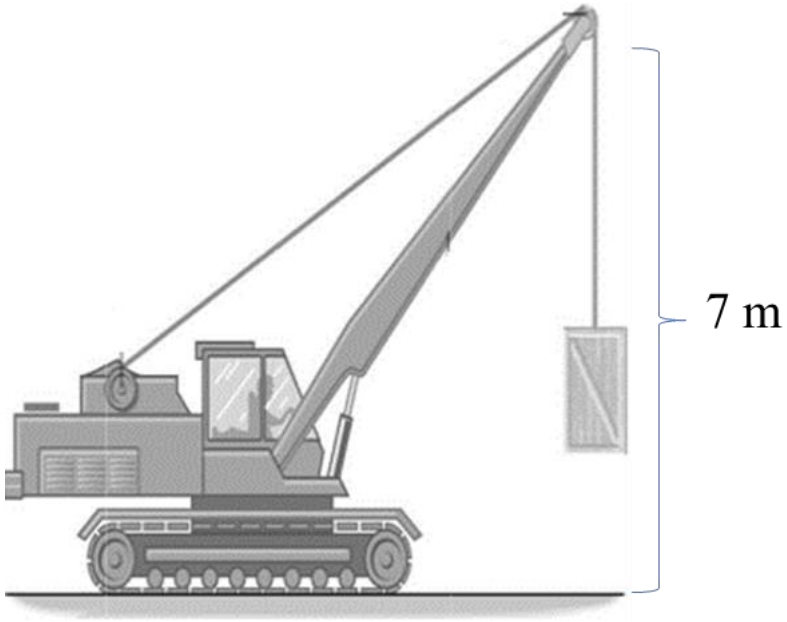
P is the instantaneous power

F is the force acting on the object

v is the instantaneous velocity of the object

θ is the angle between $\vec{F}$ and $\vec{v}$

Example: a crane takes 3.5 s to lift a box of 1500 N for a vertical distance 7 m. (a) what is the work done by the crane during this lift? (b) what is the average power of the crane during this lift?



Solution:

a)

$$W = \vec{F} \cdot \vec{d}$$
$$= Fd\ cos\theta$$
$$= 1500 \times 7 \times \cos 0°$$
$$= 10500\ \text{J}$$

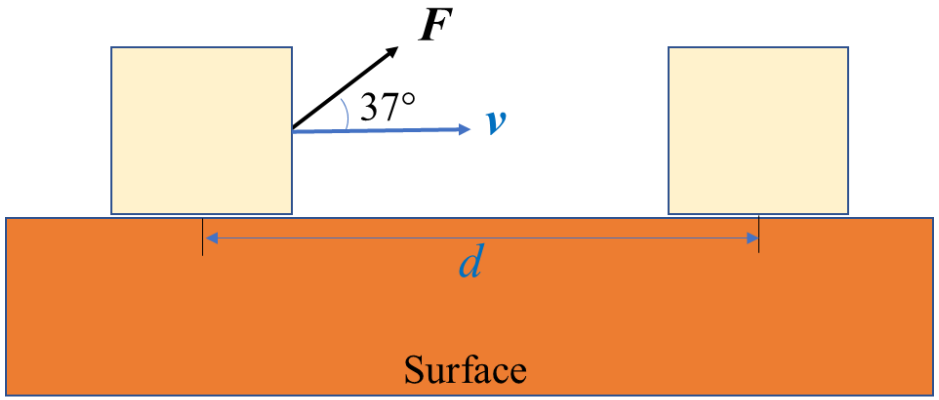
b)

$$P_{avg} = \frac{W}{\Delta t}$$

$$= \frac{10500}{3.5}$$

$$= 3000\ \text{W}$$

Example: A 100 kg block is pulled at a constant speed of 5 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which work is done on the block by this force?



Solution:

Remember: the rate at which work is done = power

$$P = \vec{F} \cdot \vec{v}$$

$$= Fv\cos\theta$$

$$= 122 \times 5 \times \cos 37°$$

$$= 487.2\ \text{W}$$

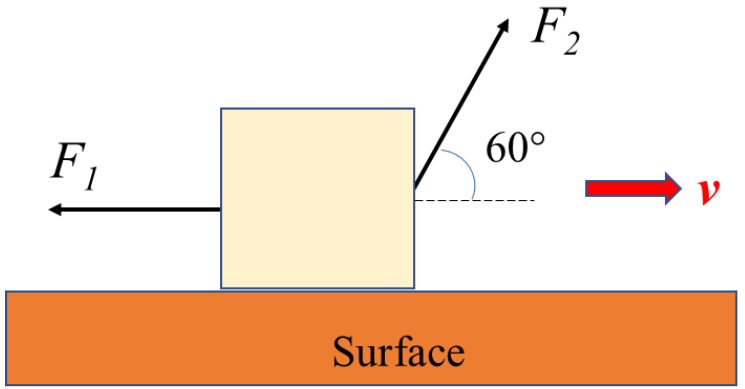
Note:

$\because$ speed is constat $\Longrightarrow$ $v = v_{avg}$

$\Longrightarrow$ $P = P_{avg}$

$\therefore$ we can use the lows of the instantaneous power or the average power to solve this example.

Example: the figure shows constant forces F_1 and F_2 acting on a box as the box slides rightward across a frictionless floor. Force F_1 is horizontal, with magnitude 2 N; force F_2 is angled upward by 60° to the floor and has magnitude 4 N. The speed v of the box at a certain instant is 3 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?



Solution:

Power of F_1: $P_1 = F_1 v\, cos\theta = 2 \times 3 \times \cos 180° = -6$ W

Power of F_2: $P_2 = F_2 v\, cos\theta = 4 \times 3 \times \cos 60° = 6$ W

Net power: $P_{net} = P_1 + P_2$

$= -6 + 6 = 0$ W

which tells us that there is no net power at this instant ⟹ no net work is done on the box ⟹ transfer of energy to or from the box is zero ⟹ the speed of the box will remain at 3 m/s

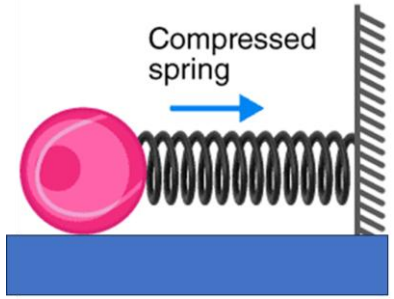
Chapter 8

Potential Energy and Conservation of Energy

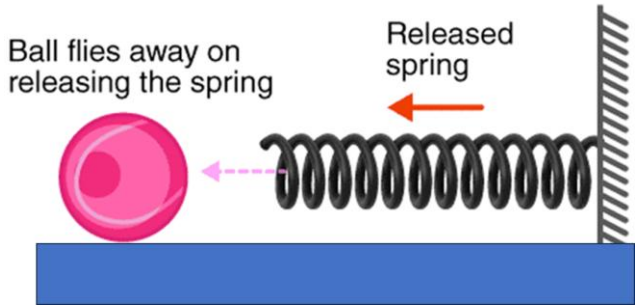
Potential Energy

- The potential energy is the energy that the objects obtain because of their position or configuration (arrangement).
- Potential energy is "stored" in the object, and can transfer to kinetic energy.

For example, a spring stores potential energy when it is compressed, and then transfers this potential energy to kinetic energy when it is released.

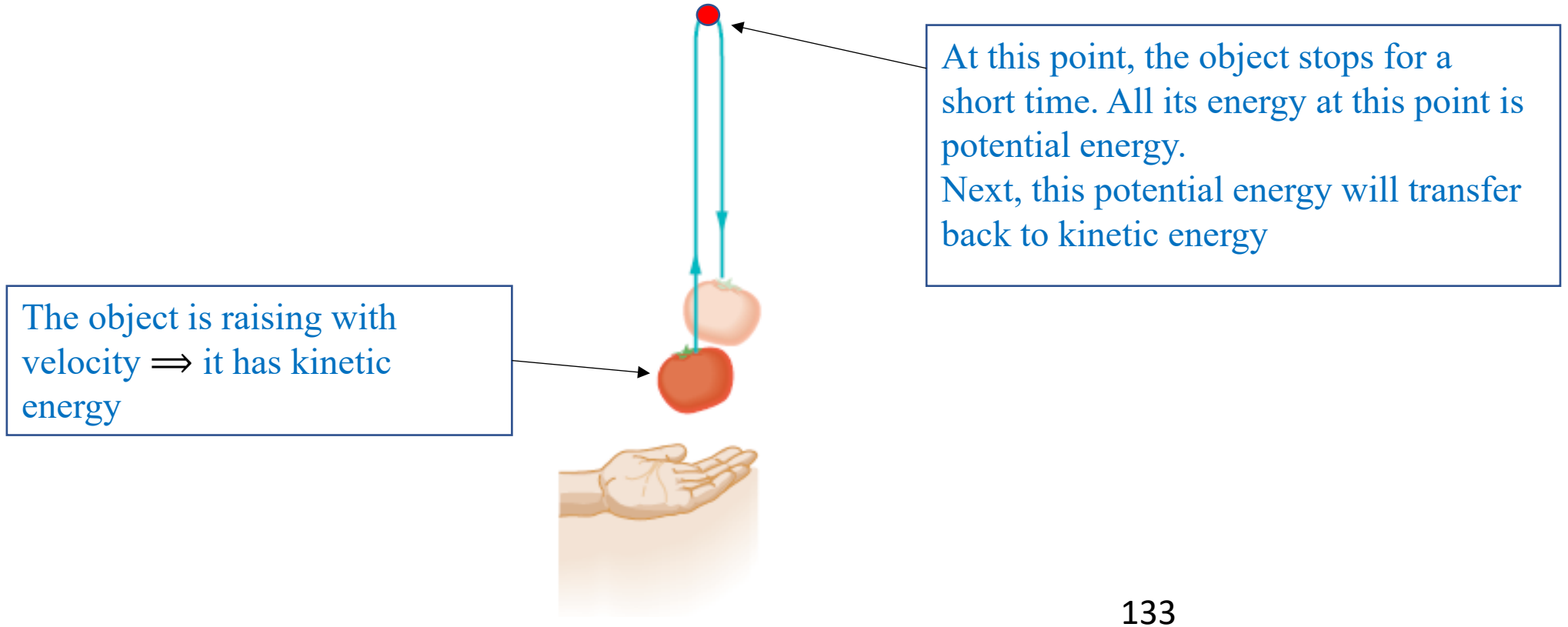


The compressed spring stores potential energy now (because of its configuration). Both the ball and the spring are at rest (no kinetic energy)



When the spring is released, all the stored potential energy transfer to kinetic energy ⟹ the ball now is moving (has also kinetic energy)

- Also, when you throw an object upward, you do work on the object ⟹ the object now has velocity (and kinetic energy)
- The velocity will decrease (less kinetic energy) until the object stops at the maximum height
- Now, all the kinetic energy has transferred to potential energy (because of the position of the object)
- Next, the object will move downward (increase in kinetic energy and decrease in potential energy)



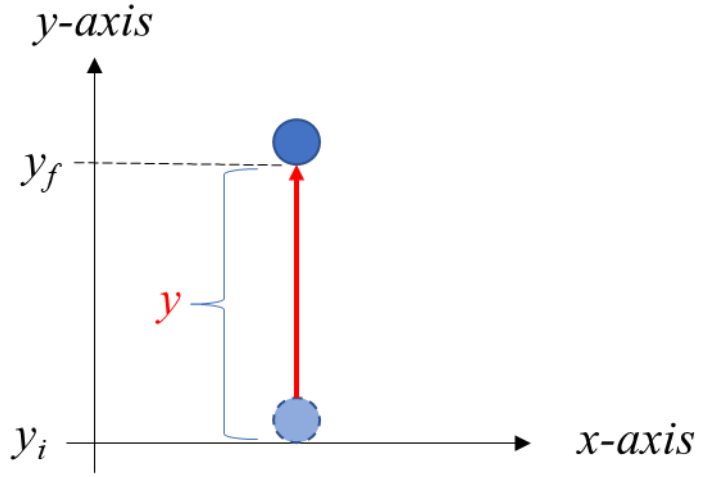
Gravitational Potential Energy

- When you move an object upward from the ground (in the *y-axis* direction), this object gains potential energy U
- Assume you left the object from height y_i to another height y_f
- You can calculate the gravitational potential energy U from the equation:

$$U = m\,g\,(y_f - y_i)$$

or simply

$$U = m\,g\,y$$



where,
U is the gravitational potential energy
m is the mass of the object
g is the gravitational acceleration
y is the height that the object moved

Remember: All energies (kinetic and potential) are scalar quantities, and have the SI unit of Joule (J)

- **Note** that in the equation: $U = m\,g\,y$

  The height y must be measured from a ***reference point***

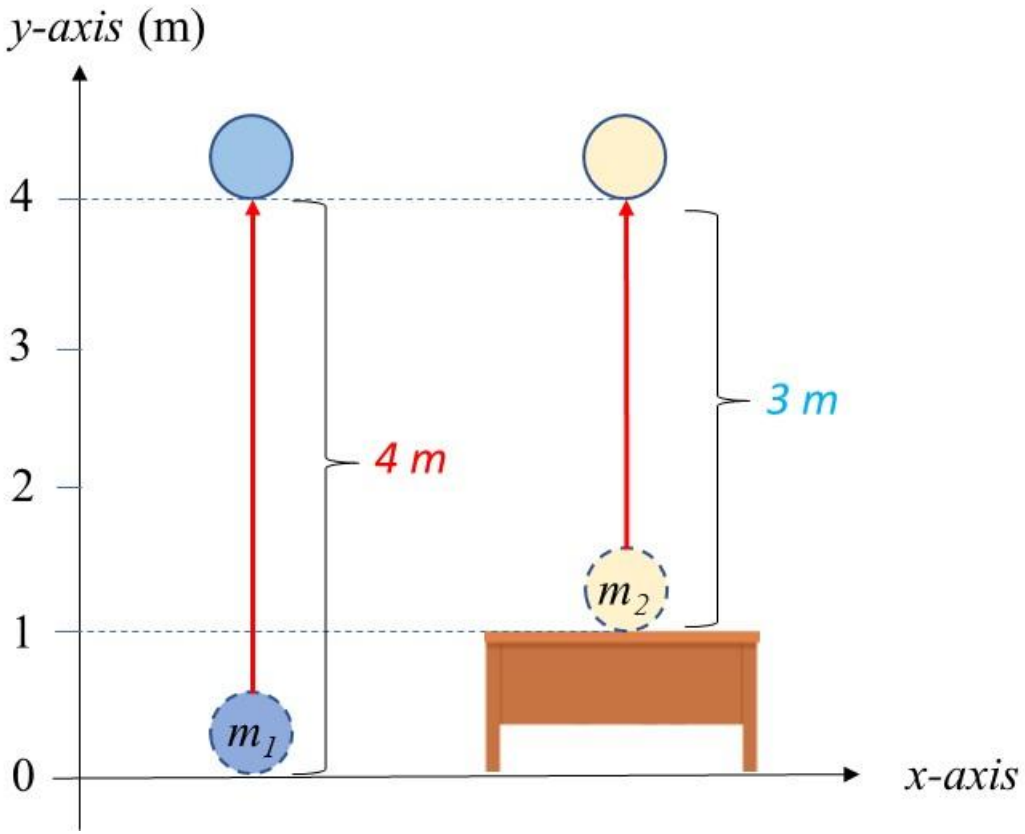
- For example, mass $m_1 = 5$ kg is placed on the ground, and another identical mass $m_2 = 5$ kg is placed on a table as shown in the figure. They are lifted to the same level on the y-axis.

- U for m_1 will be: $U = m_1\,g\,\mathbf{y} = 5 \times 9.8 \times \mathbf{4} = 196$ J

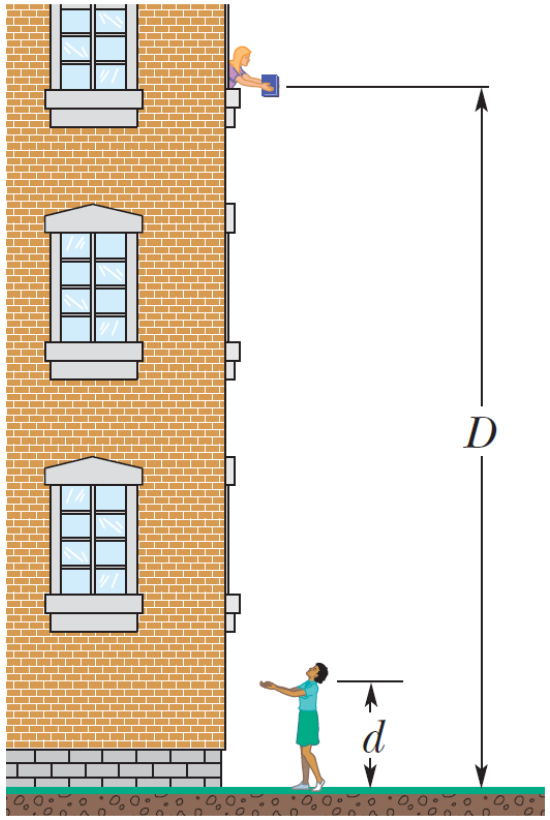
  BUT

- U for m_2 will be: $U = m_2\,g\,y = 5 \times 9.8 \times 3 = 147$ J

- Although both masses are lifted to the same level, their potential energies are different because each mass has a different reference point.



Example: You drop a 2 kg book to a friend who stands on the ground at distance $D = 10$ m below. If your friend's outstretched hands are at distance $d = 1.50$ m above the ground. What is the change ΔU in the gravitational potential energy of the book–Earth system during the drop?



Solution:
The potential energy at the distance D is:

$$U_i = m\,g\,D$$
$$= 2\times 9.8 \times 10 = 196 \text{ J}$$

The potential energy at the distance d is:

$$U_f = m\,g\,d$$
$$= 2\times 9.8 \times 1.5 = 29.4 \text{ J}$$

The change in the potential energy:

$$\Delta U = U_f - U_i$$
$$= 29.4 - 196 = -166.6 \text{ J}$$

This means that the book is loosing gravitational potential energy as it falls down

Conservation of Mechanical Energy

The **mechanical energy** E_{mec} of an object is equal to the sum of its potential energy U and the kinetic energy K:

$$E_{mec} = K + U$$

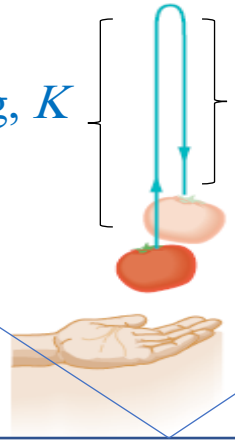
In this part, we study only the systems with the following conditions:

1- *no friction forces* in our systems

2- the system is *isolated* from its environment; that is, *NO external force* from outside the system causes energy changes inside the system

In these systems, the total mechanical energy E_{mec} of the system is constant

When the object is raising, K decreases but U increases



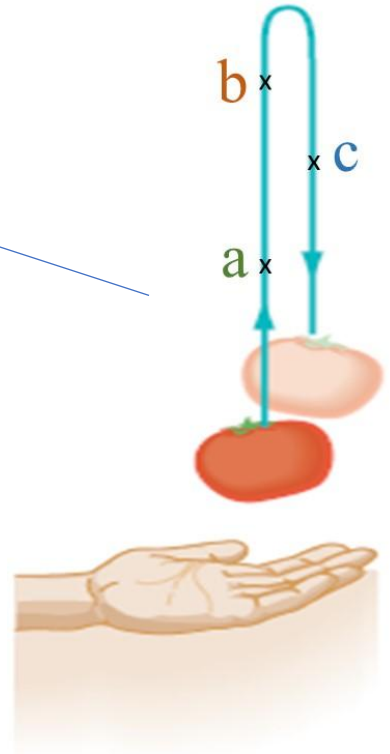
When the object is falling, K increases but U decreases

In both cases and at each point of the object's path, the sum $K+U$ is always the same

For example, if you choose several points in the path of the object (a, b and c), you will find that the sum of K and U at any point is exactly equal to the sum of K and U at any other point, which equal to the total E_{mec} of the object.

That is,

$$K_a + U_a = K_b + U_b = K_c + U_c = E_{mec}$$



- ∴ In general, if there is a system that changes from *state 1* to *state 2*:

$$E_{mec,1} = E_{mec,2}$$

$$\begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for any} \\ \text{state of a system}\end{pmatrix} = \begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for any other} \\ \text{state of the system}\end{pmatrix}$$

$$K_1 + U_1 = K_2 + U_2$$

- This means that the kinetic energy alone and potential energy alone can change, but their sum, the mechanical energy E_{mec} of the system cannot change.
- This result is called the **principle of conservation of mechanical energy**.

∵ the mechanical energy cannot change, this means:

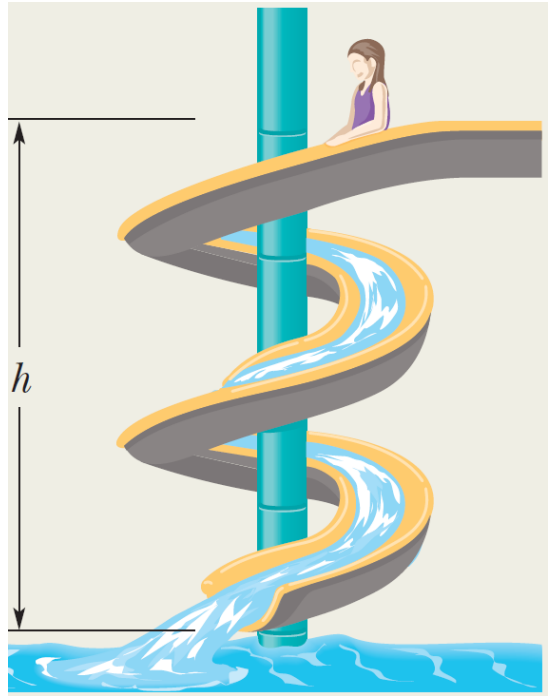
$$\Delta E_{mec} = 0$$

but we can see that,

$$\Delta E_{mec} = \Delta K + \Delta U$$

$$\therefore \quad \Delta E_{mec} = \Delta K + \Delta U = 0$$

Example: a child of mass *m* is released from rest at the top of a water slide, at height *h* = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



Solution:

The mechanical energy is conserved at the top $E_{mec,t}$ and the bottom $E_{mec,b}$ of the slide:

$$E_{mec,b} = E_{mec,t}$$

$$K_b + U_b = K_t + U_t$$

$$\frac{1}{2}mv_b^2 + mgy_b = \frac{1}{2}mv_t^2 + mgy_t$$

Dividing by *m*:

$$\frac{1}{2}v_b^2 + gy_b = \frac{1}{2}v_t^2 + gy_t$$

$$\frac{1}{2}v_b^2 = \frac{1}{2}v_t^2 + gy_t - gy_b$$

$$\frac{1}{2}v_b^2 = \frac{1}{2}v_t^2 + g(y_t - y_b)$$

The initial velocity at the top = 0 $\Longrightarrow$ $v_t = 0$, and $(y_t - y_b) = h$:

$$\frac{1}{2}v_b^2 = g(h)$$

$$v_b = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 8.5} = 12.9 \text{ m/s}$$

Work Done on a System by an External Force

- In the last part we learned how to deal with ***isolated systems*** (no external force or work was applied from outside the system (where $\Delta E_{mec} = 0$)

- Now, we will study the systems that have some *external forces acting* on them (*not isolated systems*)
- In these systems: $\Delta E_{mec} \neq 0$

- In Chapter 7, we learned that:
  - **Work** ***W*** is equal to the energy transferred to or from an object by means of a force acting on the object.
  - Energy transferred **to** the object is **positive** work.
  - Energy transferred **from** the object is **negative** work.

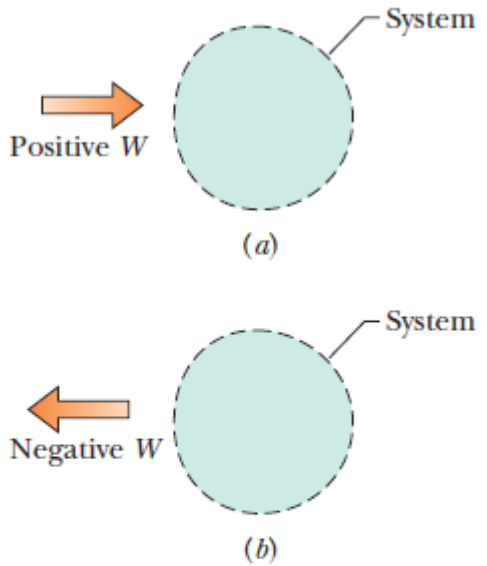
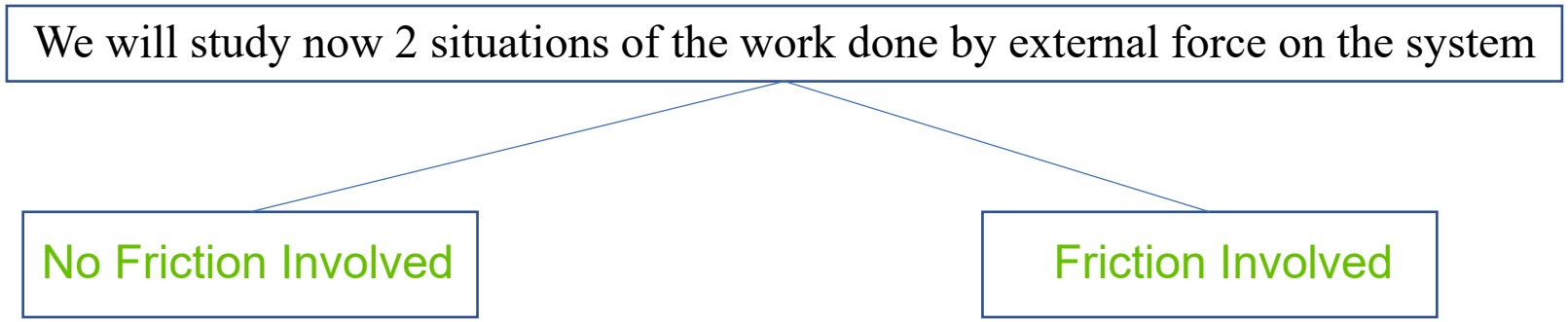


Figure 8-11 (*a*) Positive work *W* done on an arbitrary system means a transfer of energy to the system. (*b*) Negative work *W* means a transfer of energy from the system.

∴ in the system with external forces, we must consider the work *W* done on the system when we calculate E_{mec}


We will study now 2 situations of the work done by external force on the system
No Friction Involved
Friction Involved

No Friction Involved

- If there is work W done on a system, this will change the mechanical energy E_{mec} of the system $\Longrightarrow$ E_{mec} is NOT constant any more

- If there is NO friction force, then the work W done on the system must equal the change in its mechanical energy E_{mec}

$$W = \Delta E_{mec}$$

or

$$W = \Delta K + \Delta U$$

- This means that any work W you do on a system, must change to K or U.

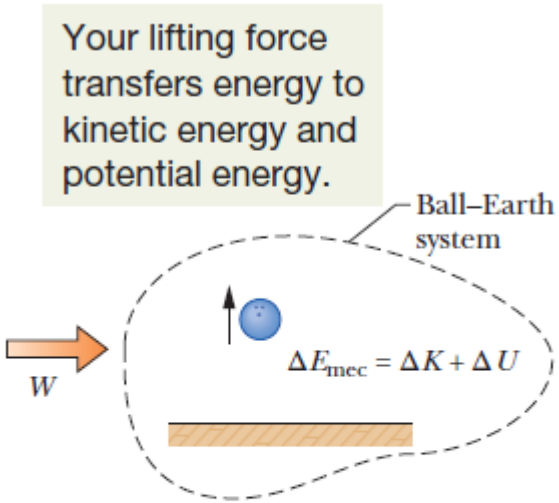
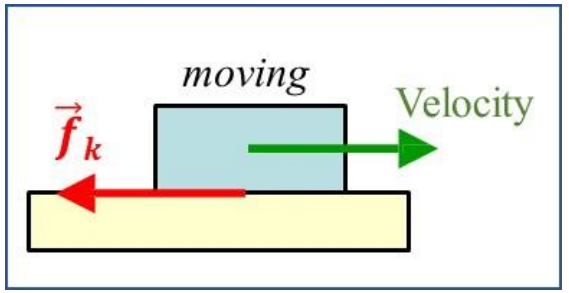


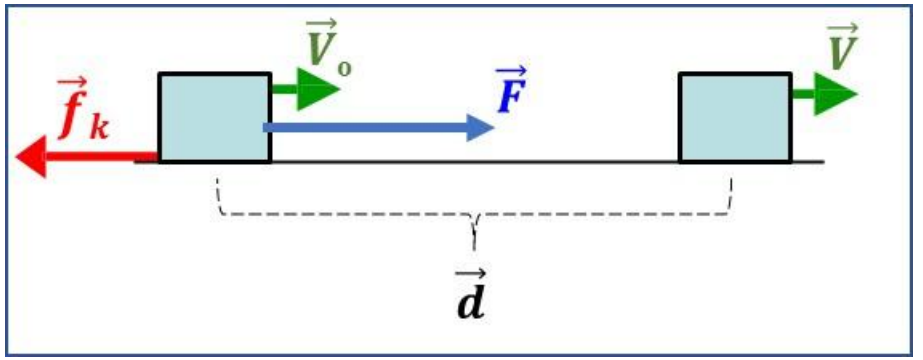
Figure 8-12 Positive work W is done on a system of a bowling ball and Earth, causing a change ΔE_{mec} in the mechanical energy of the system, a change ΔK in the ball's kinetic energy, and a change ΔU in the system's gravitational potential energy.

Friction Involved



- We learned that is an object is moving on a rough surface, there is a friction force $\boldsymbol{f_k}$ acting on the object.
- Also, we learned that $\vec{f}_k = \mu_k F_N$

- If an object moves as shown in the figure, then the friction appears as "heat" or thermal energy E_{th} in the system, and can be calculated from:



$$E_{th} = f_k\, d$$

where,
E_{th} is the thermal energy produced by friction
f_k is the frictional force
d is the displacement of the object

- The work W done by the external force $\vec{F}$ is:

$$W = \Delta E_{mec} + \Delta E_{th}$$

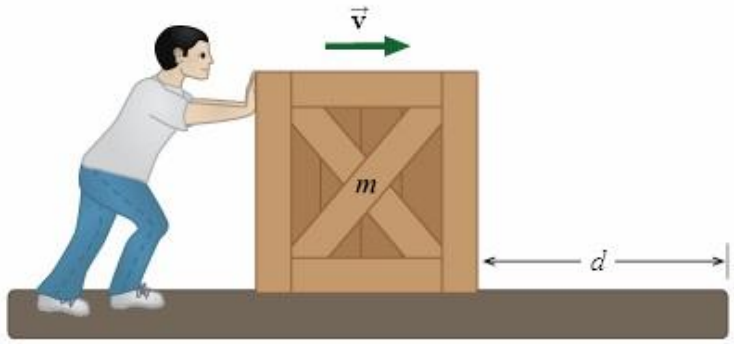
$$W = \Delta K + \Delta U + \Delta E_{th}$$

- This means that any work you do on a system, must change to K or U and heat (E_{th})

Summary

System type	Equation
Isolated (no external force)	$\Delta E_{mec} = 0$ $\Delta E_{mec} = \Delta K + \Delta U$
Not isolated (external force but no friction)	$W = \Delta E_{mec}$ $W = \Delta K + \Delta U$ (and still $\Delta E_{mec} = \Delta K + \Delta U$)
Not isolated (external force but with friction)	$W = \Delta E_{mec} + \Delta E_{th}$ $W = \Delta K + \Delta U + \Delta E_{th}$

Example: A man pushes a wood box of mass $m = 14$ kg across the floor with a constant horizontal force F of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.5$ m, the speed of the box decreases from $v_0 = 0.6$ m/s to $v = 0.2$ m/s. (a) How much work is done by force F? (b) What is the increase ΔE_{th} in the thermal energy of the box and floor? (c) Calculate the frictional force between the box and the floor.



Solution:

a) $W = Fd\cos\theta = 40 \times 0.5 \times \cos 0° = 20$ J

b) ∵ the box is slowing down ⟹ there is friction between the box and the floor (the system in not isolated, and has friction)
∴ to calculate the change in the mechanical energy we use:

$$W = \Delta E_{mec} + \Delta E_{th}$$

Remember that: $\Delta E_{mec} = \Delta K + \Delta U$
but there is no change in the potential energy ⟹ $\Delta U = 0$

$$\Longrightarrow \Delta E_{mec} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$$

$$\therefore\ W = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 + \Delta E_{th}$$

$$W = \frac{1}{2}m(v^2 - v_o^2) + \Delta E_{th}$$

$$\Delta E_{th} = W - \frac{1}{2}m(v^2 - v_o^2)$$
$$= 20 - \frac{1}{2} \times 14\left(0.2^2 - 0.6^2\right)$$
$$= 22.2 \text{ J}$$

c) $E_{th} = f_k\, d$

$$f_k = \frac{E_{th}}{d}$$
$$= \frac{22.2}{0.5}$$
$$= 44.4 \text{ N}$$

Power

- We learned in Chapter 7 that power P is related to work W as:

$$P_{avg} = \frac{W}{\Delta t} \quad \text{and} \quad P = \frac{dW}{dt}$$

- and also, we know that: work  energy

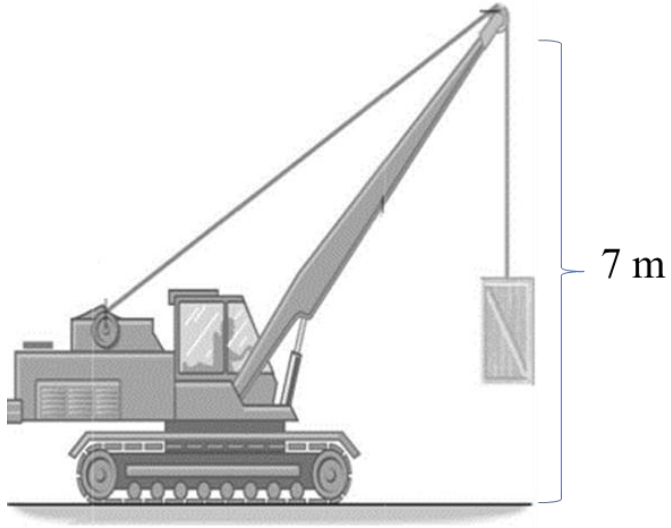
- ∴ we can define the average power as:

$$\boxed{P_{avg} = \frac{\Delta E}{\Delta t}}$$

and the instantaneous power as:

$$\boxed{P = \frac{dE}{dt}}$$

Example: a crane takes 3.5 s to lift a box of a mass of 153 kg for a vertical distance 7 m. What is the average power of the crane during this lift? Assume that there is no friction.



Solution:

- The crane applies external force on the box during lifting the box over the ground

$\therefore$ the system (box and ground) is not isolated, and has no friction:

$W = \Delta K + \Delta U$ or $\Delta E_{mec} = \Delta K + \Delta U$

- The box is stationary at the ground and also will be stationary when it reaches the 7m height $\Longrightarrow$ the change in its kinetic energy $\Delta K = 0$,

$\therefore \Delta E_{mec} = \Delta U$

$$\begin{aligned}\Delta E_{mec} &= \Delta U \\ &= m g y_f - m g y_i \\ &= m g (y_f - y_i) \\ &= mgh \\ &= 153 \times 9.8 \times 7 \\ &= 10496 \text{ J}\end{aligned}$$

Now we can calculate the power:

$$P_{avg} = \frac{\Delta E}{\Delta t}$$

$$= \frac{10496}{3.5}$$

$$\approx 3000 \text{ W}$$