# Statistics and Probabilities for Computing (CEN1602)

Unit-3

## Discrete Random Variables and Probability Distributions

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# Outline

**1. Discrete Random Variables**
- Definition
- Mapping outcomes to numerical values

**2. Probability Distributions**
- Probability Mass Function (PMF)
- PMF properties
- Tabular and graphical forms

**3. Cumulative Distribution Function (CDF)**
- Definition
- Relationship to PMF
- Using CDF to compute probabilities

**4. Mean and Variance**
- Expected value
- Variance and standard deviation

**5. Discrete Uniform Distribution**
- Definition and assumptions
- Mean and variance

**6. Binomial Distribution**
- Bernoulli trials
- PMF
- Mean and variance

**7. Geometric Distribution**
- Trials until first success
- PMF, mean, and variance

**8. Negative Binomial Distribution**
- Trials until $r$-th success
- PMF, mean, and variance

**9. Hypergeometric Distribution**
- Sampling without replacement
- PMF, mean, and variance

**10. Poisson Distribution**
- Modeling event counts in fixed intervals
- PMF
- Mean and variance

# Learning Objectives for Unit 3

**By the end of this unit, students should be able to:**

1. Define a discrete random variable and explain how it represents outcomes of a random experiment using numbers.
2. Construct and interpret a Probability Mass Function (PMF).
3. Use the Cumulative Distribution Function (CDF) to calculate probabilities.
4. Compute the expected value (mean) of a discrete random variable and explain its meaning.
5. Compute the variance and standard deviation and explain what they measure.
6. Identify the conditions of a binomial experiment and apply the binomial distribution.
7. Distinguish between geometric, negative binomial, and hypergeometric distributions.
8. Use the Poisson distribution to model the number of events occurring in a fixed interval.

# Discrete Random Variables

- A **random variable** is a function that assigns a numerical value to each outcome of a random experiment.
- It allows us to study randomness using numbers instead of listing outcomes.
- A random variable is usually written using a capital letter (for example, $X$).
- A **discrete random variable** takes a finite or countable number of possible values.
- These values are often whole numbers (non-negative integers) because they represent counts.
- Each possible value of a discrete random variable **has its probability**.
- The set of all possible values of the random variable is called its **range**.

# Example: Packet Transmission

In a computer network, data are sent in small units called **packets**. Each packet can either:

- Arrive successfully (S), or
- Be lost (L)




If a device sends **3** packets. Assume:

- Probability of success, $P(S) = 0.8$
- Probability of loss, $P(L) = 0.2$
- Packets are independent

The sample space is:

SSS, SSL, SLS, LSS, SLL, LSL, LLS, LLL

Each outcome has probability determined by multiplying 0.8 (success) and 0.2 (loss) as needed. Example:

$$P(SSL) = (0.8)(0.8)(0.2) = 0.128$$

Let $X$ be a **discrete random variable** that represents the number of successfully received packets. The possible values of $X$ are: 0, 1, 2, 3

Now we can group outcomes by the value of $X$and construct its **discrete probability distribution**.

# Discrete Probability Distribution

- A **discrete probability distribution** shows how probabilities are assigned to the possible values of a discrete random variable.
- For a discrete random variable $X$:
  - Each value of $X$ has a probability.
  - The sum of all probabilities equals 1.
- The **function** that gives these probabilities for a discrete random variable is called the **Probability Mass Function (PMF)**.
- **PMF** is written as $p(x) = P(X = x)$ , meaning the probability that the random variable $X$ takes the value $x$.
- Probability Distribution of the previous Packet Transmission Example is:

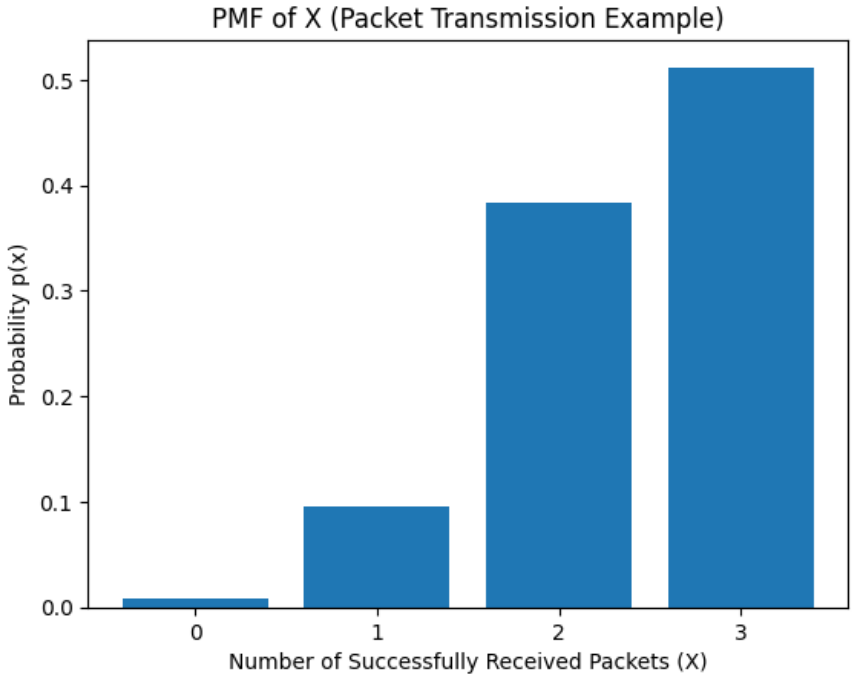
PMF Table

| $x$ | Included outcomes | How $p(x)$ is computed | $p(x)$ |
|---|---|---|---|
| 0 | LLL | $(0.2)$^3 | 0.008 |
| 1 | SLL, LSL, LLS | $3(0.8)(0.2^2)$ | 0.096 |
| 2 | SSL, SLS, LSS | $3(0.8^2)\ (0.2)$ | 0.384 |
| 3 | SSS | $0.8^3$ | 0.512 |

- The table shows the probability assigned to each possible value of the random variable.
- The probabilities of all possible values add up to 0.008+0.096+0.384+0.512=1.

# Graphical Representation of the PMF

- The probability mass function (PMF) of $X$ can be represented using a bar chart.
- Each bar represents a possible value of $X$, and the height of the bar for $X = x$ equals $p(x)$.
- For the packet transmission example, the bar chart is shown here.




- What this Graph Shows?
  - Taller bars represent more likely values.
  - The most likely value is $X = 3$.
  - The total area (sum of heights) equals 1.

# Cumulative Distribution Function (CDF)

- The **Cumulative Distribution Function (CDF)** gives the probability that a random variable is **less than or equal** to a given value.
- For a discrete random variable $X$, the CDF is defined as:

$$\boldsymbol{F(x) = P(X \leq x) = \sum_{x_i \leq x} p(x_i)}$$

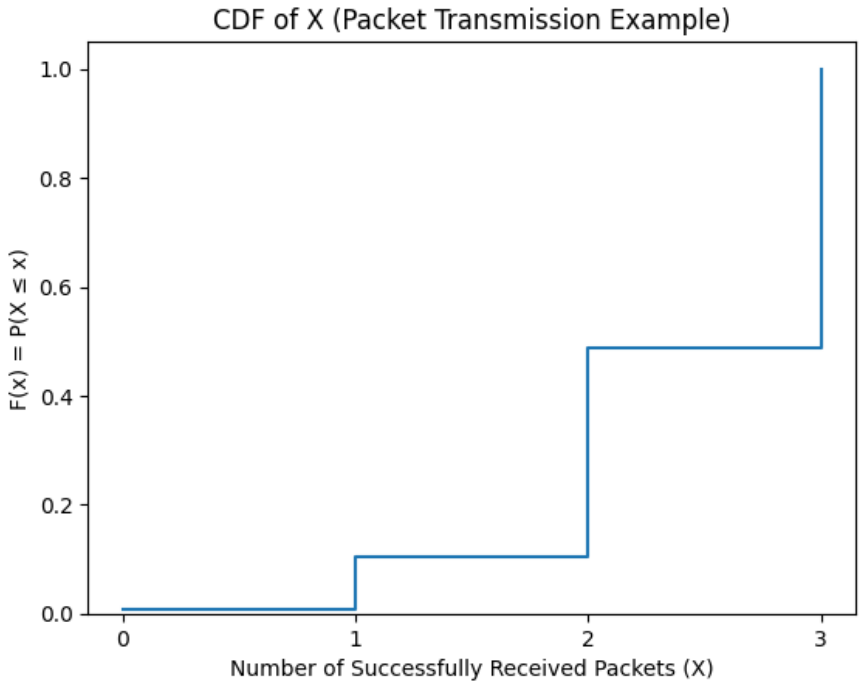
- To compute $F(x)$, we add the probabilities of all values less than or equal to $x$.
- The CDF is obtained from the PMF by cumulative addition.
- The CDF is always non-decreasing (it never decreases).
- The last value of the CDF equals 1 because the total probability is 1.
- The CDF helps us compute probabilities such as:

$$P(X \leq 2),\ P(X > 1),\ \text{and}\ P(a < X \leq b)$$

# CDF for Packet Transmission Example

CDF Table

| $x$ | $F(x) = P(X \leq x)$ |
|---|---|
| 0 | 0.008 |
| 1 | 0.008 + 0.096 = 0.104 |
| 2 | 0.104 + 0.384 = 0.488 |
| 3 | 0.488 + 0.512 = 1.000 |




- This clearly shows:
  - The CDF increases step-by-step.
  - It jumps at integer values only (since X is discrete).
  - The final value reaches 1.

# Using the CDF

- The CDF allows us to compute probabilities using cumulative values without summing many PMF values.
- To find $P(X \leq a)$ we directly use $F(a)$.
- To find $P(X > a)$, we use the complement rule:

$$P(X > a) = 1 - P(X \leq a) = 1 - F(a)$$

- To find $P(a < X \leq b)$, we subtract cumulative values:

$$P(a < X \leq b) = F(b) - F(a)$$

**Example (Packet Transmission)**

- $P(X \leq 2) = F(2) = 0.488$
- $P(X > 1) = 1 - F(1) = 1 - 0.104 = 0.896$
- $P(1 < X \leq 3) = F(3) - F(1) = 1 - 0.104 = 0.896$

CDF Table

| $x$ | $F(x)$ |
|---|---|
| 0 | 0.008 |
| 1 | 0.104 |
| 2 | 0.488 |
| 3 | 1.000 |

# Mean (Expected Value) of a Discrete Random Variable

- The mean of a discrete random variable is the average value we expect if the experiment is repeated many times.
- It is also called the **expected value** and is written as $E(X)$.
- For a discrete random variable,

$$\boldsymbol{E(X) = \sum x\, p(x)}$$

- To calculate it, multiply each value of $X$ by its probability, then add the results.
- For the packet example, the values and probabilities are: 0, 1, 2, 3 with probabilities 0.008, 0.096, 0.384, 0.512.

Thus,

$$E(X) = 0(0.008) + 1(0.096) + 2(0.384) + 3(0.512) = 2.4$$

- This means that, on average, about **2.4 packets** are successfully received out of 3.

# Variance and Standard Deviation

- Variance measures how spread out the values of a random variable are from the mean.
- If the values are close to the mean, the variance is small. If the values are far from the mean, the variance is large.
- For a discrete random variable, the variance is defined as:

$$\boldsymbol{Var(X) = \sum (x - \mu)^2 p(x)}$$

where $\mu = E(X)$

- A simpler formula for calculation is:

$$\boldsymbol{Var(X) = E(X^2) - [E(X)]^2}, \text{ where } E(X^2) = \sum x^2 \; p(x)$$

- The standard deviation is the square root of the variance:

$$\boldsymbol{\sigma = \sqrt{Var(X)}}$$

- Variance and standard deviation tell us how much variation exists around the average value.

# Example: Waiting Time for Shuttle Bus Near Al-Haram

- During moderate traffic, the waiting time (in minutes) for a shuttle bus follows the probability distribution below.

| $X$ (minutes) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $p(x)$ | 0.15 | 0.20 | 0.25 | 0.20 | 0.15 | 0.05 |

- **Mean (Expected Value):** $E(X) = \sum x\ p(x)$

$$E(X) = 1(0.15) + 2(0.20) + 3(0.25) + 4(0.20) + 5(0.15) + 6(0.05)$$
$$E(X) = 0.15 + 0.40 + 0.75 + 0.80 + 0.75 + 0.30 = 3.15 \textit{ minutes}$$

- **Compute** $E(X^2)$:

$$E(X^2) = 1^2(0.15) + 2^2(0.20) + 3^2(0.25) + 4^2(0.20) + 5^2(0.15) + 6^2(0.05)$$
$$E(X^2) = 0.15 + 0.80 + 2.25 + 3.20 + 3.75 + 1.80 = 11.95$$

- **Variance:**

$$Var(X) = E(X^2) - [E(X)]^2$$
$$\text{Var}(X) = 11.95 - (3.15)^2 = 11.95 - 9.9225 = 2.0275$$

- **Standard Deviation:**

$$\sigma = \sqrt{2.0275} \approx 1.42 \textit{ minutes}$$

- **Interpretation (simple):**
  - Average waiting time ≈ **3.15 minutes**
  - Typical variation ≈ **1.42 minutes** around the average

# Discrete Uniform Distribution

- A **discrete uniform distribution** means all outcomes have the **same probability**.
- If $X$ can take $n$ values, then each value has probability:

$$p(x) = \frac{1}{n}$$

- **Example**: rolling a fair die (1 to 6), each value has probability $\frac{1}{6}$ .
- The PMF is flat: all bars in the PMF bar chart have the same height.
- **Mean (Expected Value)**: For any discrete uniform distribution with $n$ distinct outcomes $x_1, x_2, \dots, x_n$), the expected value is:

$$\boldsymbol{E[X] = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

- **Variance**: For a set of $n$ equally likely outcomes $x_1, x_2, \dots, x_n$:

$$\boldsymbol{Var(X) = \frac{\sum(x_i - \mu)^2}{n}}$$

Where $\mu$ is the expected value (mean).

# Example: Cloud Data Storage Region Selection

A company stores user backups in one of six data centers located at the following distances from Makkah (in km) to balance load and redundancy, each location is equally likely:

$$50, 200, 400, 800, 1200, 1500$$

To Let $X =$ *distance (km) of selected data center*

1. Find the expected value $E(X)$.
2. Find the variance $Var(X)$
3. Find the standard deviation.
4. Interpret the results.

- This is a discrete uniform distribution with $p(x) = \frac{1}{6}$.
- The expected value is $E(X) = \frac{50+200+400+800+1200+1500}{6} = \mathbf{691.67\ km}$.
- This means that, on average, user data is stored about 692 km away.
- Using the variance formula, $\mathrm{Var(X)} = \frac{\sum(x-691.67)^2}{6} \approx \mathbf{264,444\ km}$.
- The standard deviation is $\sigma \approx \mathbf{514\ km}$.
- The large standard deviation shows that storage locations are widely spread.
- In practical, engineers use the mean to estimate the average network delay. They use the standard deviation to measure how widely the data centers are spread, which helps improve reliability and backup safety.

# Binomial Distribution

- The **binomial distribution** models the number of successes in repeated trials.
- A random variable $X$ follows a binomial distribution if:
  1. The number of trials $n$is fixed.
  2. Each trial has only two outcomes (success or failure).
  3. The probability of success $\boldsymbol{p}$ is the same in every trial.
  4. The trials are independent.
- When these conditions are satisfied, the trials are called **Bernoulli trials**.

**Example 1:** A device sends 10 packets. Each packet is either received (success) or lost (failure).
$X$= number of successfully received packets.

**Example 2:** A user tries to log in 6 times. Each attempt is either correct (success) or incorrect (failure).
$X$= number of successful logins.

**Example 3:** A tester checks 6 devices. Each device either passes (success) or fails (failure).
$X$= number of devices that pass.

# Binomial Probability Mass Function (PMF)

- If a random variable $X$ follows a binomial distribution. Then $P(X = x)$, the probability of $x$ successes out of $n$ trials is given by:

$$\boldsymbol{P(X = x) = \binom{n}{x} p^x q^{n-x}}$$

Where,

- $n$ = total number of trials.
- $x$ = number of successes.
- $p$ = probability of success in one trial.
- $q = 1 - p$ = probability of failure.
- $\binom{n}{x}$ counts the number of different ways to get $x$ successes out of $n$.

**Example:** Toss a fair coin 8 times. Find the probability of getting exactly 3 heads.

$X$ is the number of heads after n trials.
Since $n = 8, p = 0.5, q = 0.5$, then

$$P(X = 3) = \binom{8}{3}(0.5)^3(0.5)^5 = 0.219.$$

# Mean and Variance of Binomial Distribution

- For a binomial random variable, the mean (expected value) is:

$$\boldsymbol{E(X) = np}$$

- This represents the average number of successes.
- The variance is:

$$\boldsymbol{Var(X) = np(1-p) = npq}$$

- The standard deviation is:

$$\boldsymbol{\sigma = \sqrt{Var(X)} = \sqrt{npq}}$$

- The mean depends on $n$ and $p$.
- The spread depends on both the probability of success and failure.

# Example: Defective Products in a Factory

A factory produces electronic chips. Each chip has a 2% probability of being defective. An inspector checks 150 random chips. Let $X$= number of working chips.

1. What is the expected number of working chips?
2. What is the variance?
3. What is the standard deviation?
4. What do these values mean in practice?

$$p = 0.98, \qquad q = 0.02, \qquad n = 150$$

- **Expected Value:** $E(X) = np = 150(0.98) = 147$

  On average, **147 chips** are working.

- **Variance:** $\mathrm{Var}(X) = npq = 150(0.98)(0.02) = 2.94$

- **Standard Deviation:** $\sigma = \sqrt{2.94} \approx 1.715$

- **Practical meaning:**
  The factory expects about 147 working chips per 150 tested.
  The standard deviation shows that the number of working chips usually varies by about 1-2 chips. If variability increases, the production process may have problems.

# Example: Vaccine Effectiveness Study

A new vaccine is tested on 50 people. Historical data shows that the vaccine works with probability $p = 0.9$.

1. What is the expected number of people who develop immunity?
2. What is the variance?
3. What is the standard deviation?
4. What do these values mean in practice?

$$p = 0.9, \qquad q = 0.1, \qquad n = 50$$

- **Expected Value:** $E(X) = np = 50(0.9) = 45$

  On average, **45 people** will develop immunity.

- **Variance:** $\mathrm{Var}(X) = npq = 50(0.9)(0.1) = 4.5$

- **Standard Deviation:** $\sigma = \sqrt{4.5} \approx 2.12$

- **Practical Meaning:**

  The health administration expects about 45 successful cases. The standard deviation (≈2) shows that results typically vary by about 2 people. If only 38 people develop immunity, this is far below expectation and needs investigation.

  These numbers help evaluate whether the vaccine is performing as expected.

# Geometric Distribution

- The **geometric distribution** is closely similar to the binomial distribution.
- In the binomial distribution:
  - The number of trials is fixed = $n$.
  - **We count how many successes happen in $n$ trials.**
- In the geometric distribution:
  - The number of trials is **not fixed**.
  - We continue performing trials **until the first success happens**.
  - **We count how many trials are needed to get that first success.**
- The key difference:
  - **Binomial** → number of successes in fixed trials.
  - **Geometric** → number of trials until first success.

**Example 1:** A fair coin is tossed repeatedly until the first head appears.

$X$= number of tosses needed to get the first head.

**Example 2:** A user tries to log in, and each attempt has probability $p = 0.7$ of success. The user continues trying until the first successful login.

$X$= number of attempts needed to log in successfully.

# Geometric Distribution – PMF

- If $X$ is a **geometric** random variable with probability of success $p$, then:

$$P(X = x) = (1 - p)^{x-1}p = q^{x-1}p$$

- Explanation:
  - The first $x - 1$ trials must be failures.
  - The $x$-th trial must be the first success.

  So the probability becomes:

$$(\text{failure})^{x-1} \times (\textit{success})$$

**Example:** A worker is checking spare batteries in a storage box. Each battery has a 10% probability of being defective. The worker checks batteries one by one until the first **working** battery is found. What is the probability that the first working battery is found on the **third check**?

since $q = 0.1,\ p = 0.9$

$$P(X = 3) = q^{3-1}p$$
$$P(X = 3) = (0.1)^2(0.9)$$
$$P(X = 3) = 0.009$$

# Geometric Distribution – Mean & Variance

If $X$ is a **geometric** random variable with probability of success $p$, then:

- The mean (expected value) is:

$$\boldsymbol{E(X) = \frac{1}{p}}$$

- The mean represents the average number of trials needed to get the first success.
- The variance is:

$$\boldsymbol{\mathrm{Var}(X) = \frac{1-p}{p^2} = \frac{q}{p^2}}$$

- The standard deviation is:

$$\boldsymbol{\sigma = \sqrt{\frac{q}{p^2}}}$$

# Example: Searching for an AB− Blood Donor

AB negative (AB−) is a rare blood type found in about 2% of the population. The probability that a randomly tested person has AB− blood is $p = 0.02$. People are tested one by one until the first AB− donor is found.

Let: $X =$ *number of people tested until the first AB− match*

1. What is the expected number of people tested?
2. What is the variance?
3. What is the standard deviation?
4. What do these values mean in practice?

**1. Expected Value:** $E(X) = \frac{1}{p} = \frac{1}{0.02} = 50$

On average, **50 people** must be tested to find one match.

**2. Variance:** $\mathrm{Var}(X) = \frac{q}{p^2} = \frac{0.98}{0.0004} = 2450$

**3. Standard Deviation** $\sigma = \sqrt{2450} \approx 49.5$

**4. Practical Meaning**

- The mean (50) helps estimate how many tests are usually required.
- The large standard deviation shows high uncertainty in search time.
- Sometimes a donor is found quickly; sometimes many tests are needed.
- This helps plan the best medical staffing and emergency response.

# Negative Binomial Distribution

- The **negative binomial distribution** is an extension of the geometric distribution.
  - Geometric distribution: number of trials until the **first success** ($r = 1$).
  - Negative binomial distribution: number of trials until the ***r*-th success**.
- When $r = 1$, the negative binomial distribution becomes the geometric distribution.
- The probability mass function is:

$$P(X = x) = \binom{x-1}{r-1} p^r q^{x-r}$$

- The mean (expected value) is: $E(X) = \frac{r}{p}$
- The variance is: $Var(X) = \frac{rq}{p^2}$
- The standard deviation: $\sigma = \sqrt{\frac{rq}{p^2}}$

# Negative Binomial Distribution

A technician is testing used rechargeable batteries. Each battery works properly with probability $p = 0.6$ . The technician needs **4 working batteries** for a device. Batteries are tested one by one until 4 working ones are found.

Let X=number of batteries tested until 4 working batteries are found

1. What is the probability that exactly **7 batteries** are tested to obtain 4 working ones?
2. What is the expected value of $X$?
3. What is the variance?
4. What is the standard deviation?

**1. PMF Calculation:** $P(X = 7) = \binom{6}{3}(0.6)^4(0.4)^3 \approx 0.166$

**2. Expected Value:** $E(X) = \frac{r}{p} = \frac{4}{0.6} = 6.67$

On average, about **7 batteries** must be tested.

**3. Variance:** $\mathrm{Var}(X) = \frac{rq}{p^2} = \frac{4(0.4)}{0.36} = 4.44$

**4. Standard Deviation:** $\sigma = \sqrt{4.44} \approx 2.11$

Functional Meaning:

- The PMF tells how likely a specific testing effort (7 batteries) is.
- The mean helps estimate average required inventory.
- The standard deviation shows how much the testing effort may vary.
- This helps decide how many extra batteries should be kept available.

# Hypergeometric Distribution

- The **hypergeometric distribution** models the number of successes obtained when selecting a sample **without replacement** from a finite population.
- **Parameters:**
  $\boldsymbol{N}$ = Total population size
  $\boldsymbol{K}$ = Number of successes in the population
  $\boldsymbol{n}$ = Sample size (number selected)
  $\boldsymbol{x}$ = Number of successes in the sample
- **Remember**: without replacement means any item in the population cannot be selected more than once.
- Unlike binomial distribution:
  - **Hypergeometric** → probability changes after each draw.
  - **Binomial** → probability stays constant.

- **Example:** A shipment contains **12 network cables (**5 defective and 7 working). A technician randomly selects **4 cables** for testing. Let $X =$ *number of working cables selected*. What is the probability that **3 working cables** are selected?

  Here "success" = working cable, $\boldsymbol{N}$ **= 12** (population size), $\boldsymbol{K}$ **= 7** (Number of working cables), $\boldsymbol{n}$ **= 4** (Number selected), $\boldsymbol{x}$**=3** (Number of working cables in the sample)

# Hypergeometric PMF

If $X$= number of successes in a sample drawn **without replacement**, then:

$$\boldsymbol{P(X=x)=\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}}$$

**For previous example (Success = Working cables):**

- $N=12$
- $K=7$
- $n=4$
- Find $P(X=3)$:

$$P(X=3)=\frac{\binom{7}{3}\binom{5}{1}}{\binom{12}{4}}$$

$$P(X=3)=\frac{35\times 5}{495}=\frac{35}{99}\approx 0.354$$

# Hypergeometric Mean and Variance

- **Expected Value (Mean):**

$$\boldsymbol{E(X) = n\left(\frac{K}{N}\right)}$$

- **Variance:**

$$\boldsymbol{Var(X) = n\left(\frac{K}{N}\right)\left(1 - \frac{K}{N}\right)\left(\frac{N-n}{N-1}\right)}$$

- **Standard Deviation:**

$$\boldsymbol{\sigma = \sqrt{Var(X)}}$$

**Apply to Our Example (Working Cables):**

- **Mean:** $E(X) = 4\left(\frac{7}{12}\right) = \frac{28}{12} = 2.33$
- **Variance:** $Var(X) = 4\left(\frac{7}{12}\right)\left(\frac{5}{12}\right)\left(\frac{12-4}{12-1}\right) = \frac{1120}{1584} \approx 0.707$
- Standard deviation: $\sigma = \sqrt{Var(X)} \approx \sqrt{0.707} \approx 0.84$

# Poisson Distribution

The **Poisson distribution** models the number of events occurring in a fixed interval of time, space, or area.

**Examples:**

- Number of phone calls received in one hour
- Number of cars passing a checkpoint in 10 minutes
- Number of packet losses per 100 km of fiber optic cable
- Number of signal errors per kilometer of transmission line
- Number of dead pixels per square inch of a screen
- Number of hardware defects per silicon wafer area
- Number of network requests per second
- Number of packets arriving in 1 millisecond

Use **Poisson** when:

1. We count events in a **fixed interval**
2. Events occur randomly
3. Events occur independently
4. The **average rate** is constant. We denote the average rate by: $\lambda$

# Poisson PMF

$$\boldsymbol{P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}}$$

where $e \approx 2.718$

**Example 1:** A web server receives on average **3 requests per second**. What is the probability that exactly **5 requests arrive in one second?**

Since $\lambda = 3$ and $x = 5$:

$$P(X = 5) = \frac{e^{-3}3^{5}}{5!} \approx 0.1008$$

**Example 2**: A fiber optic cable experiences on average **3 transmission errors per 100 km**. What is the probability that exactly 4 transmission errors occur along 100 km of cable.

Since $\lambda = 3$ and $x = 4$:

$$P(X = 4) = \frac{e^{-3}3^{4}}{4!} = 0.168$$

# Poisson Distribution: Expected Value, Variance, and Standard Deviation

- **Expected Value (Mean):**

$$\boldsymbol{E(X) = \lambda}$$

The mean equals the average rate of occurrence.

- **Variance:**

$$\boldsymbol{Var(X) = \lambda}$$

- **Standard Deviation:**

$$\boldsymbol{\sigma = \sqrt{\lambda}}$$

**Example:** A bank receives on average **8 customers per hour**. Let $X$=number of customers arriving in one hour

1. What is $E(X)$?
2. What is $Var(X)$?
3. What is the standard deviation?

1. $E(X) = 8$
2. $Var(X) = 8$
3. $\sigma \approx 2.83$

Most hours will have roughly between **5 and 11 customers** (8 ± 2.83).

# Core Ideas Summary for Unit-3

- A discrete random variable assigns numerical values to outcomes of a random experiment.
- The probability mass function (PMF) describes how probability is distributed across possible values.
- The cumulative distribution function (CDF) gives $P(X \leq x)$
- The expected value represents the long-run average outcome.
- Variance and standard deviation measure variability around the mean.
- Special discrete distributions model different situations:
  - Discrete Uniform: equal probabilities
  - Binomial: fixed number of independent trials
  - Geometric: number of trials until the first success
  - Negative Binomial: number of trials until the $r$-th success
  - Hypergeometric: counting successes in a sample taken from a population
  - Poisson: counting events in time, space, or area
- Selecting the correct distribution depends on:
  - Fixed trials or not
  - Replacement or no replacement
  - Counting successes or waiting for them
  - Events occurring in intervals